

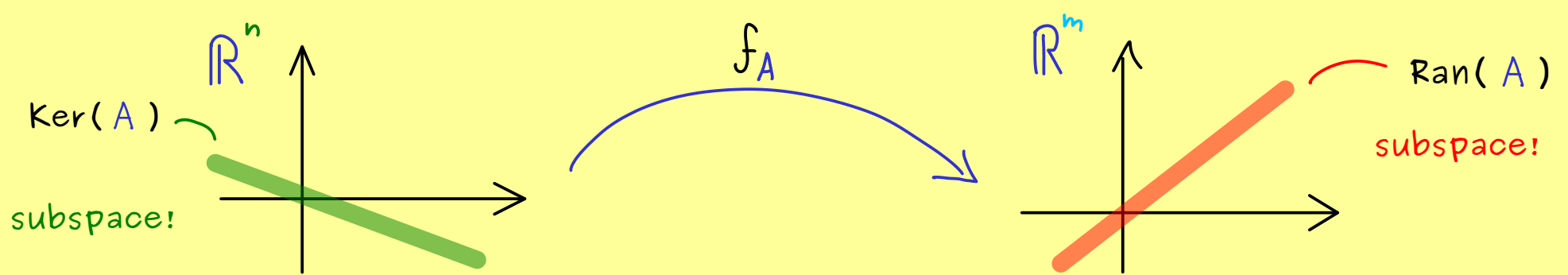


## Linear Algebra - Part 34

$A \in \mathbb{R}^{m \times n}$  induces a linear map  $f_A: \mathbb{R}^n \rightarrow \mathbb{R}^m, x \mapsto Ax$

$$\begin{aligned} \text{Ran}(A) &:= \{Ax \mid x \in \mathbb{R}^n\} \subseteq \mathbb{R}^m && \text{range of } A \text{ (image of } A) \\ &\equiv \text{Ran}(f_A) && \text{(see Start Learning Sets - Part 5)} \end{aligned}$$

$$\begin{aligned} \text{Ker}(A) &:= \{x \in \mathbb{R}^n \mid Ax = 0\} \subseteq \mathbb{R}^n && \text{kernel of } A \\ &\equiv f_A^{-1}[\{0\}] && \text{preimage of } \{0\} \text{ under } f_A \\ &&& \text{(nullspace of } A) \end{aligned}$$



Remember:  $\text{Ran}(A) = \text{span}(a_1, a_2, \dots, a_n)$        $A = \begin{pmatrix} | & & | \\ a_1 & \dots & a_n \\ | & & | \end{pmatrix}$

Solving LES?  $Ax = b$       existence of solutions:  $b \in \text{Ran}(A)$  ?  
 uniqueness of solutions:  $\text{Ker}(A) \neq \{0\}$  ?