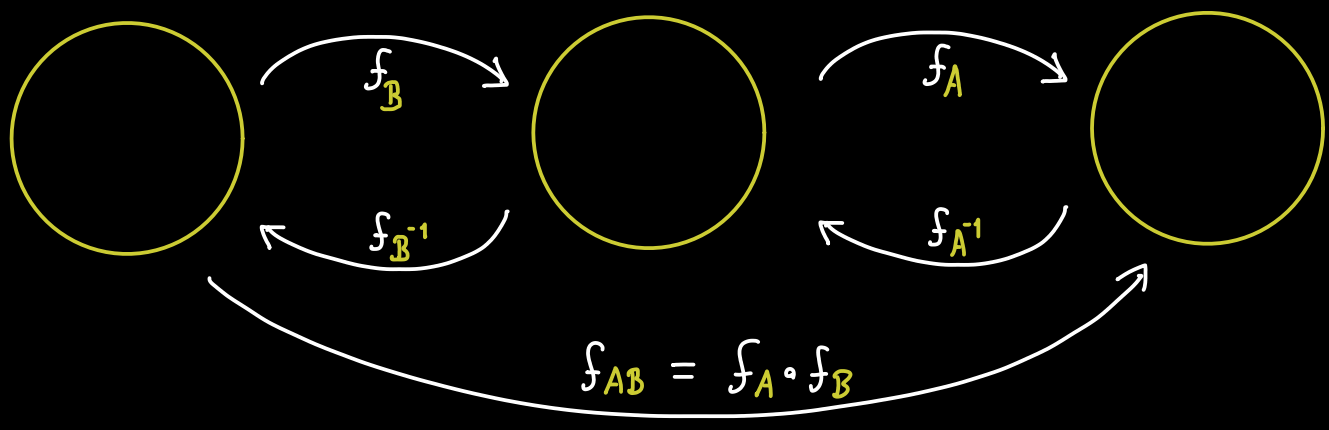
Linear Algebra – Part 31

matrices $A, B \in \mathbb{R}^{n \times n}$



We have: $f_{B^{-1}} \circ f_{A^{-1}} = (f_{AB})^{-1} \Rightarrow \boxed{(AB)^{-1} = B^{-1}A^{-1}}$

Important fact: $f: \mathbb{R}^n \longrightarrow \mathbb{R}^n$ linear and bijective

$\Rightarrow f^{-1}: \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is also linear

Proof:

$$f^{-1}(\lambda y) = f^{-1}(\lambda \cdot f(x)) \underbrace{= f^{-1}(f(\lambda x))}_{f \text{ linear}} = \lambda \cdot x = \lambda f^{-1}(y) \quad \checkmark$$

There is exactly one x with $f(x) = y$

$$f^{-1}(y + \tilde{y}) = f^{-1}(f(x) + f(\tilde{x})) \underbrace{= f^{-1}(f(x + \tilde{x}))}_{f \text{ linear}} = x + \tilde{x}$$

$$= f^{-1}(y) + f^{-1}(\tilde{y}) \quad \checkmark$$