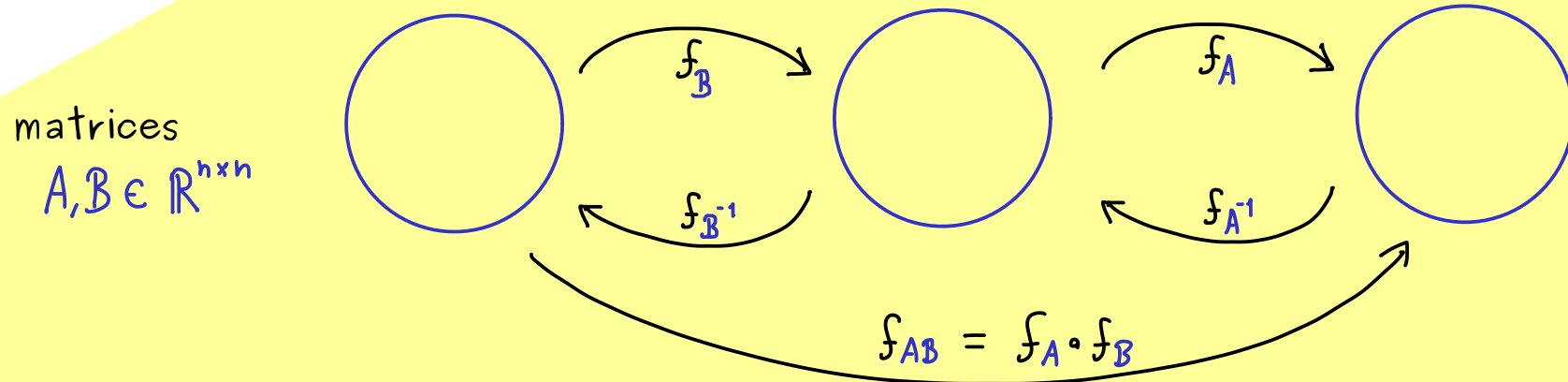




Linear Algebra - Part 31



We have: $f_B^{-1} \circ f_A^{-1} = (f_{AB})^{-1} \Rightarrow (AB)^{-1} = B^{-1}A^{-1}$

Important fact: $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ linear and bijective

$$\Rightarrow f^{-1}: \mathbb{R}^n \rightarrow \mathbb{R}^n \text{ is also linear}$$

Proof: $f^{-1}(\lambda y) = f^{-1}(\lambda \cdot f(x)) = f^{-1}(\underbrace{f(\lambda x)}_{f \text{ linear}}) = \lambda \cdot x = \lambda f^{-1}(y) \checkmark$

There is exactly one x with $f(x) = y$

$$\begin{aligned} f^{-1}(y + \tilde{y}) &= f^{-1}(f(x) + f(\tilde{x})) = f^{-1}(\underbrace{f(x + \tilde{x})}_{f \text{ linear}}) = x + \tilde{x} \\ &= f^{-1}(y) + f^{-1}(\tilde{y}) \checkmark \end{aligned}$$