Linear Algebra - Part 27

Steinitz Exchange Lemma: $\left(v^{(1)}, v^{(2)}, \dots, v^{(k)}\right)$ basis of U

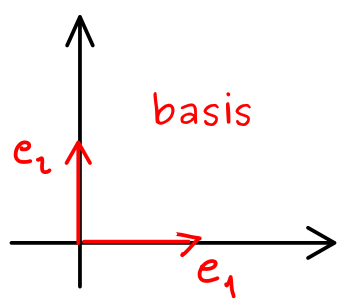
$\left(a^{(1)}, a^{(2)}, \dots, a^{(\ell)}\right)$ lin. independent vectors in U

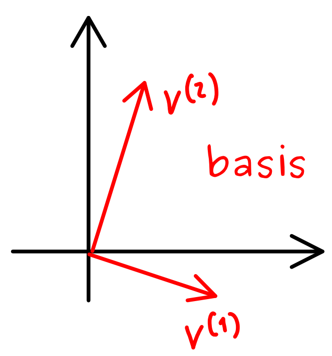
$\Longrightarrow$ new basis of U

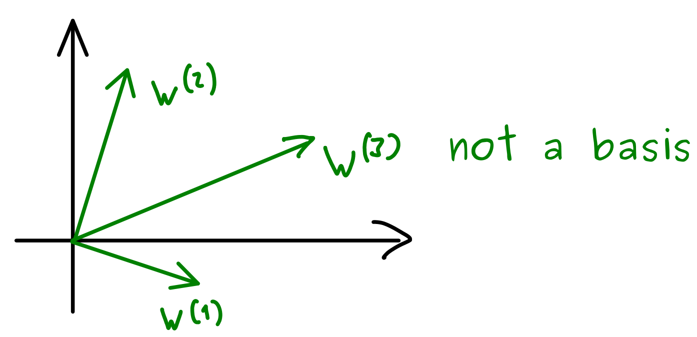
Fact: Let $U \subseteq \mathbb{R}^n$ be a subspace and $\mathcal{B} = \left(v^{(1)}, v^{(2)}, \dots, v^{(k)}\right)$ be a basis of U.

Then: (a) Each family $\left(w^{(1)}, w^{(2)}, \dots, w^{(m)}\right)$ with $m > k$ vectors in U is linearly dependent.

(b) Each basis of U has exactly k elements.







Definition: Let $U \subseteq \mathbb{R}^n$ be a subspace and $\mathcal{B}$ be a basis of U.

The number of vectors in $\mathcal{B}$ is called the dimension of U.

We write: $\dim(U)$ integer

Set: $\dim(\{0\}) := 0$ $\qquad \left(\operatorname{Span}(\emptyset) = \{0\} \right)$ basis

Example: $(e_1, e_2, \dots, e_n)$ standard basis of $\mathbb{R}^n$

$$\dim(\mathbb{R}^n) = n$$