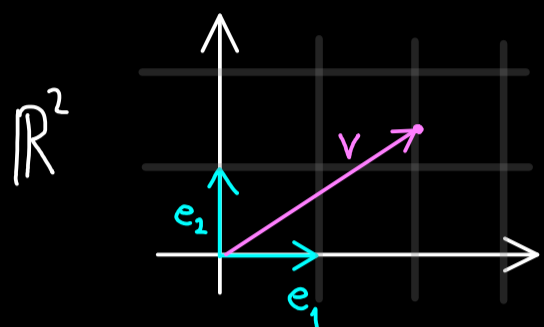
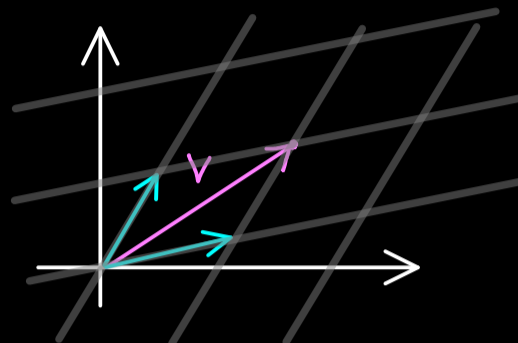


Linear Algebra - Part 25

basis of a subspace: spans the subspace + linearly independent



$$v = \begin{pmatrix} 2 \\ \frac{4}{3} \end{pmatrix}$$



coordinates of v :

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

coordinates: $U \subseteq \mathbb{R}^n$ subspace, $\mathcal{B} = (v^{(1)}, v^{(2)}, \dots, v^{(k)})$ basis of U

\Rightarrow Each vector $u \in U$ can be written as a linear combination:

$$u = \lambda_1 v^{(1)} + \lambda_2 v^{(2)} + \dots + \lambda_k v^{(k)}, \quad \lambda_j \in \mathbb{R} \text{ (uniquely determined)}$$

coordinates of u with respect to \mathcal{B}

$$u = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_k \end{pmatrix}_{\mathcal{B}}$$

Example: $\mathbb{R}^3 = \text{span} \left(\begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \right)$
basis of \mathbb{R}^3

$$u = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = 1 \cdot \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix} + 2 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

$$\tilde{u} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} = -1 \cdot \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$