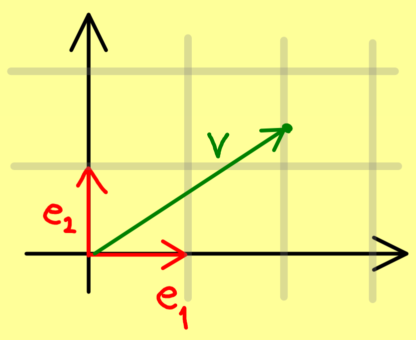
The Bright Side of Mathematics

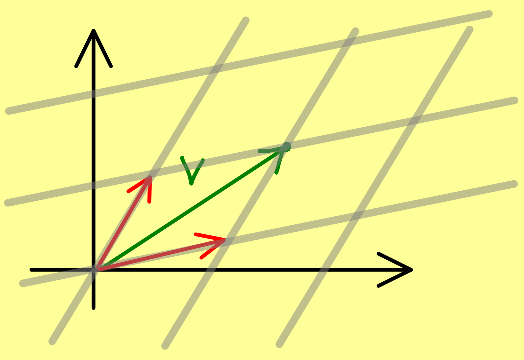
Linear Algebra – Part 25

basis of a subspace: spans the subspace + linearly independent

$\mathbb{R}^2$



$v = \begin{pmatrix} 2 \\ \frac{4}{3} \end{pmatrix}$



coordinates of v: $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

coordinates: $U \subseteq \mathbb{R}^n$ subspace, $\mathcal{B} = \left(v^{(1)}, v^{(2)}, \dots, v^{(k)}\right)$ basis of U

$\Longrightarrow$ Each vector $u \in U$ can be written as a linear combination:

$$u = \lambda_1 v^{(1)} + \lambda_2 v^{(2)} + \dots + \lambda_k v^{(k)} \quad , \quad \lambda_j \in \mathbb{R}$$

(uniquely determined)

coordinates of u with respect to $\mathcal{B}$

$$u = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_k \end{pmatrix}_{\mathcal{B}}$$

Example: $\mathbb{R}^3 = \operatorname{Span}\left(\underbrace{\begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}}_{\text{basis of } \mathbb{R}^3} \right)$

$$u = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = 1 \cdot \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix} + 2 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 1 \cdot \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

$$\tilde{u} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} = -1 \cdot \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 0 \cdot \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$