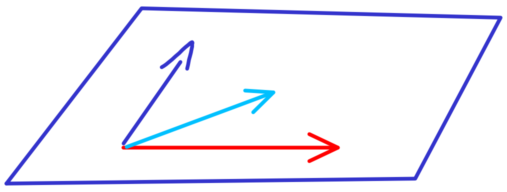
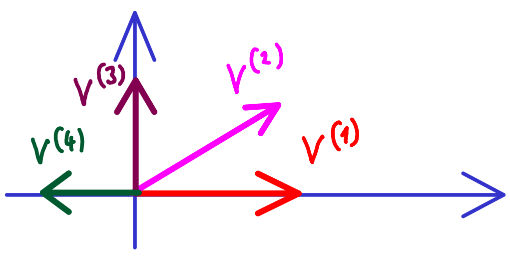
Linear Algebra - Part 24

subspace: 

$\mathcal{U} \subseteq \mathbb{R}^n$ with
(a) $0 \in \mathcal{U}$
(b) $u \in \mathcal{U}, \lambda \in \mathbb{R} \Rightarrow \lambda \cdot u \in \mathcal{U}$
(c) $u, v \in \mathcal{U} \Rightarrow u + v \in \mathcal{U}$

plane: $\mathbb{R}^2$



$$\mathrm{Span}(v^{(1)}, v^{(2)}, v^{(3)}, v^{(4)}) = \mathbb{R}^2$$

$$\mathrm{Span}(v^{(1)}, v^{(3)}) = \mathbb{R}^2$$

$$\mathrm{Span}(v^{(1)}, v^{(4)}) = \mathbb{R} \times \{0\} \neq \mathbb{R}^2$$

Definition: $\mathcal{U} \subseteq \mathbb{R}^n$ subspace, $\mathcal{B} = (v^{(1)}, v^{(2)}, \dots, v^{(k)})$, $v^{(j)} \in \mathbb{R}^n$.

$\mathcal{B}$ is called a basis of $\mathcal{U}$ if:

(a) $\mathcal{U} = \mathrm{Span}(\mathcal{B})$

(b) $\mathcal{B}$ is linearly independent

Example: $$\mathbb{R}^n = \mathrm{Span}(\underbrace{e_1, \dots, e_n}_{\text{standard basis of } \mathbb{R}^n})$$

$$\mathbb{R}^3 = \mathrm{Span}\Bigg(\underbrace{\begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}}_{\text{basis of } \mathbb{R}^3}\Bigg)$$