Linear Algebra - Part 23

$(v^{(1)}, v^{(2)}, \dots, v^{(k)})$ linearly independent if

$$\sum_{j=1}^{k} \lambda_j v^{(j)} = 0 \quad \Longrightarrow \quad \lambda_1 = \lambda_2 = \lambda_3 = \dots = 0$$

Examples:

(a) $(v^{(1)})$ linearly independent if $v^{(1)} \neq 0$

(b) $(0, v^{(2)}, \dots, v^{(k)})$ linearly dependent

$$(\lambda_1 = 1 \quad , \lambda_2 = \lambda_3 = \dots = 0)$$

(c) $\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$ linearly dependent

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$$

(d) $(e_1, e_2, \dots, e_n)$, $e_i \in \mathbb{R}^n$ canonical unit vectors

linearly independent

$$\sum_{j=1}^{n} \lambda_j e_j = 0 \quad \Longleftrightarrow \quad \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \quad \Longleftrightarrow \quad \lambda_1 = \lambda_2 = \lambda_3 = \dots = 0$$

(e) $(e_1, e_2, \dots, e_n, v)$, $e_i, v \in \mathbb{R}^n$

linearly dependent

Fact: $(v^{(1)}, v^{(2)}, \dots, v^{(k)})$ family of vectors $v^{(j)} \in \mathbb{R}^n$

linearly dependent

$\Longleftrightarrow$ There is ℓ with

$$\mathrm{Span}(v^{(1)}, v^{(2)}, \dots, v^{(k)}) = \mathrm{Span}(v^{(1)}, \dots, v^{(\ell-1)}, v^{(\ell+1)}, \dots, v^{(k)})$$