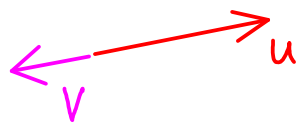
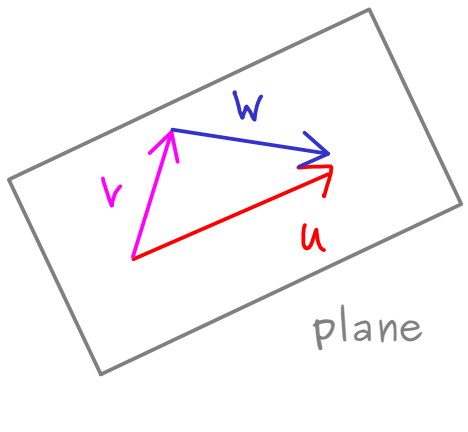
Linear Algebra – Part 22

$\mathbb{R}^2$:



colinear: $u = \lambda v$

$\mathbb{R}^3$:



coplanar: $u = \lambda v + \mu w$

$\Leftrightarrow \quad 0 = (-1)u + \lambda v + \mu w$

Definition: Let $v^{(1)}, v^{(2)}, \dots, v^{(k)} \in \mathbb{R}^n$. The family $(v^{(1)}, v^{(2)}, \dots, v^{(k)})$ (or $\{v^{(1)}, v^{(2)}, \dots, v^{(k)}\}$) is called linearly dependent if there are $\lambda_1, \lambda_2, \dots, \lambda_k \in \mathbb{R}$ that are not all equal to zero such that:

$$\sum_{j=1}^{k} \lambda_j v^{(j)} = 0$$

(zero vector in $\mathbb{R}^n$)

We call the family linearly independent if

$$\sum_{j=1}^{k} \lambda_j v^{(j)} = 0 \quad \Rightarrow \quad \lambda_1 = \lambda_2 = \lambda_3 = \dots = 0$$