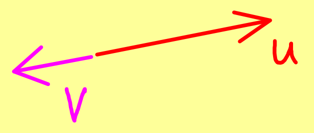
The Bright Side of Mathematics

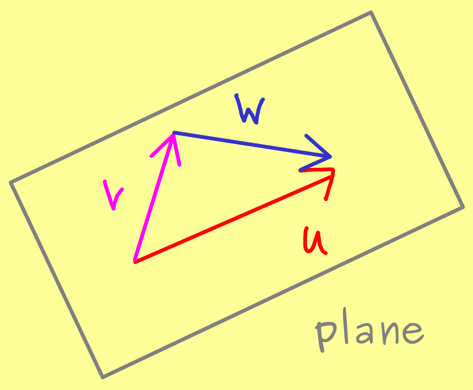
Linear Algebra – Part 22

$\mathbb{R}^2$:



colinear: $u = \lambda v$

$\mathbb{R}^3$:



coplanar: $u = \lambda v + \mu w$

$\iff 0 = (-1) u + \lambda v + \mu w$

<u>Definition:</u> Let $v^{(1)}, v^{(2)}, \dots, v^{(k)} \in \mathbb{R}^n$. The family $(v^{(1)}, v^{(2)}, \dots, v^{(k)})$ (or $\{v^{(1)}, v^{(2)}, \dots, v^{(k)}\}$) is called <u>linearly dependent</u> if there are $\lambda_1, \lambda_2, \dots, \lambda_k \in \mathbb{R}$ that are not all equal to zero such that:

$$\sum_{j=1}^{k} \lambda_j v^{(j)} = 0$$

zero vector in $\mathbb{R}^n$

We call the family <u>linearly independent</u> if

$$\sum_{j=1}^{k} \lambda_j v^{(j)} = 0 \implies \lambda_1 = \lambda_2 = \lambda_3 = \cdots = 0$$