

Linear Algebra - Part 19

$$A \in \mathbb{R}^{m \times n} \rightsquigarrow f_A: \mathbb{R}^n \rightarrow \mathbb{R}^m$$
$$x \mapsto Ax$$

Proposition: f_A is a linear map:

$$(1) \quad f_A(x+y) = f_A(x) + f_A(y) \quad , \quad A(x+y) = Ax + Ay \quad (\text{distributive})$$

$$(2) \quad f_A(\lambda \cdot x) = \lambda \cdot f_A(x) \quad , \quad A(\lambda \cdot x) = \lambda \cdot (Ax) \quad (\text{compatible})$$

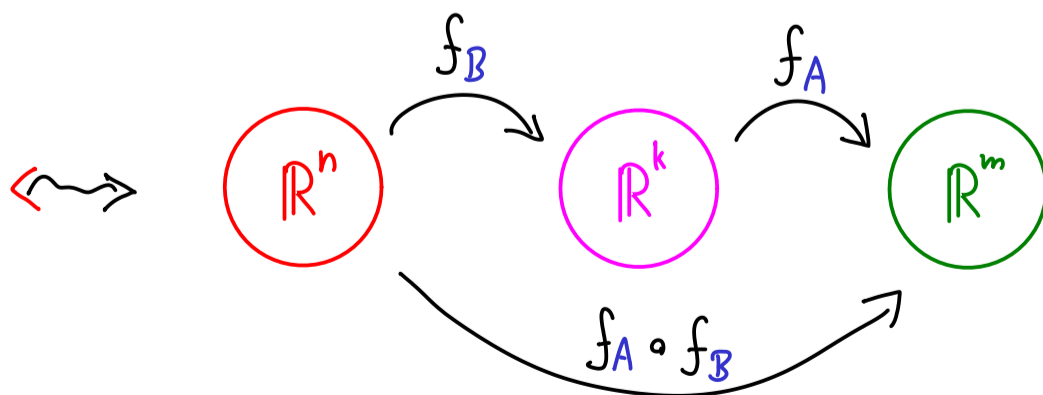
Example:

$$\begin{aligned} \begin{pmatrix} | & | \\ a_1 & a_2 \\ | & | \end{pmatrix} \left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right) &= \begin{pmatrix} | & | \\ a_1 & a_2 \\ | & | \end{pmatrix} \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix} \\ &= \begin{pmatrix} | \\ a_1 \\ | \end{pmatrix} (x_1 + y_1) + \begin{pmatrix} | \\ a_2 \\ | \end{pmatrix} (x_2 + y_2) \\ &= \begin{pmatrix} | \\ a_1 \\ | \end{pmatrix} x_1 + \begin{pmatrix} | \\ a_2 \\ | \end{pmatrix} x_2 + \begin{pmatrix} | \\ a_1 \\ | \end{pmatrix} y_1 + \begin{pmatrix} | \\ a_2 \\ | \end{pmatrix} y_2 \\ &= \begin{pmatrix} | & | \\ a_1 & a_2 \\ | & | \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} | & | \\ a_1 & a_2 \\ | & | \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \end{aligned}$$

matrix A (table of numbers) \rightsquigarrow f_A abstract linear map

Now: two matrices A, B

$$\left. \begin{array}{l} A \in \mathbb{R}^{m \times k} \\ B \in \mathbb{R}^{k \times n} \end{array} \right\} AB \in \mathbb{R}^{m \times n}$$



$$\underbrace{(f_A \circ f_B)}_{f_{AB}}(x) = f_A(f_B(x)) = f_A(Bx) = A(Bx) = (AB)x$$