Linear Algebra – Part 18

linear = conserves structure of a vector space

For the vector space $\mathbb{R}^n$: → vector addition $+$
scalar multiplication $\lambda \cdot$

Definition: $f: \mathbb{R}^n \to \mathbb{R}^m$ is called linear if for all $x, y \in \mathbb{R}^n$, $\lambda \in \mathbb{R}$:

(a) $f(\underset{\substack{\uparrow \\ \text{addition in } \mathbb{R}^n}}{x + y}) = f(x) \underset{\substack{\uparrow \\ \text{addition in } \mathbb{R}^m}}{+} f(y)$

(b) $f(\lambda \cdot x) = \lambda \cdot f(x)$

Example:

(1) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x$ linear

(2) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ not linear because $f(3 \cdot 1) = 9$
$3 \cdot f(1) = 3$ #

(3) $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x + 1$ not linear because $f(0 \cdot 1) = 1$
$0 \cdot f(1) = 0$ #