Linear Algebra – Part 16

matrix · matrix = matrix (matrix product)

$$A \in \mathbb{R}^{m \times n},\ b \in \mathbb{R}^n \rightsquigarrow Ab \in \mathbb{R}^m$$

$$A \in \mathbb{R}^{m \times n},\ b_1, \dots, b_k \in \mathbb{R}^n \rightsquigarrow Ab_1, Ab_2, \dots, Ab_k \in \mathbb{R}^m$$

$$A \cdot \underbrace{\begin{pmatrix} | & | & & | \\ b_1 & b_2 & \cdots & b_k \\ | & | & & | \end{pmatrix}}_{\in \mathbb{R}^{n \times k}} := \underbrace{\begin{pmatrix} | & | & & | \\ Ab_1 & Ab_2 & \cdots & Ab_k \\ | & | & & | \end{pmatrix}}_{\in \mathbb{R}^{m \times k}}$$

$\in \mathbb{R}^{m \times n}$

Definition: For $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times k}$, define the matrix product AB:

$$AB = \begin{pmatrix} — & \alpha_1^T & — \\ — & \alpha_2^T & — \\ & \vdots & \\ — & \alpha_m^T & — \end{pmatrix} \begin{pmatrix} | & | & & | \\ b_1 & b_2 & \cdots & b_k \\ | & | & & | \end{pmatrix} = \begin{pmatrix} \alpha_1^T b_1 & \alpha_1^T b_2 & \cdots & \alpha_1^T b_k \\ \alpha_2^T b_1 & \alpha_2^T b_2 & \cdots & \alpha_2^T b_k \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_m^T b_1 & \alpha_m^T b_2 & \cdots & \alpha_m^T b_k \end{pmatrix}$$

Example:

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 10 & 11 \end{pmatrix}$$

$$\Rightarrow AB = \begin{pmatrix} 4 & 5 \\ 10 & 11 \end{pmatrix}$$