Linear Algebra – Part 16

matrix · matrix = matrix (matrix product)

$$A \in \mathbb{R}^{m \times n}, \; b \in \mathbb{R}^{n} \;\rightsquigarrow\; Ab \in \mathbb{R}^{m}$$

$$A \in \mathbb{R}^{m \times n}, \; b_1, \dots, b_k \in \mathbb{R}^{n} \;\rightsquigarrow\; Ab_1, Ab_2, \dots, Ab_k \in \mathbb{R}^{m}$$

$$\underbrace{A}_{\in \mathbb{R}^{m \times n}} \cdot \underbrace{\begin{pmatrix} | & | & & | \\ b_1 & b_2 & \cdots & b_k \\ | & | & & | \end{pmatrix}}_{\in \mathbb{R}^{n \times k}} := \underbrace{\begin{pmatrix} | & | & & | \\ Ab_1 & Ab_2 & \cdots & Ab_k \\ | & | & & | \end{pmatrix}}_{\in \mathbb{R}^{m \times k}}$$

<u>Definition:</u> For $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times k}$, define the <u>matrix product</u> AB:

$$AB = \begin{pmatrix} \text{---} & \alpha_1^T & \text{---} \\ \text{---} & \alpha_2^T & \text{---} \\ & \vdots & \\ \text{---} & \alpha_m^T & \text{---} \end{pmatrix} \begin{pmatrix} | & | & & | \\ b_1 & b_2 & \cdots & b_k \\ | & | & & | \end{pmatrix} = \begin{pmatrix} \alpha_1^T b_1 & \alpha_1^T b_2 & \cdots & \alpha_1^T b_k \\ \alpha_2^T b_1 & \alpha_2^T b_2 & \cdots & \alpha_2^T b_k \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_m^T b_1 & \alpha_m^T b_2 & \cdots & \alpha_m^T b_k \end{pmatrix}$$

<u>Example:</u>

$$\begin{array}{cc} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} & \boxed{\begin{matrix} 4 & 5 \\ 10 & 11 \end{matrix}} \end{array} \;\Longrightarrow\; AB = \begin{pmatrix} 4 & 5 \\ 10 & 11 \end{pmatrix}$$