Linear Algebra – Part 13

Names for matrices: $A \in \mathbb{R}^{m \times n}$

m: number of rows

n: number of columns

square matrix: $A \in \mathbb{R}^{n \times n}$ for example: $\begin{pmatrix} 1 & 7 & 9 \\ 2 & 8 & 2 \\ 4 & 1 & 3 \end{pmatrix}$

column vector: $A \in \mathbb{R}^{m \times 1}$ for example: $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

row vector: $A \in \mathbb{R}^{1 \times n}$ for example: $\begin{pmatrix} 2 & 4 & 6 & 7 \end{pmatrix}$

scalar: $A \in \mathbb{R}^{1 \times 1}$ for example: (4)

diagonal matrix: $A \in \mathbb{R}^{m \times n}$, $a_{ij} = 0$ for $i \neq j$

$$\begin{pmatrix} \blacksquare & 0 & 0 & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & 0 & 0 \\ 0 & 0 & \blacksquare & 0 & 0 & 0 \\ 0 & 0 & 0 & \blacksquare & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \blacksquare & 0 & 0 \\ 0 & \blacksquare & 0 \\ 0 & 0 & \blacksquare \end{pmatrix}$$

upper triangular matrix: $A \in \mathbb{R}^{n \times n}$

$a_{ij} = 0$ for $i > j$

$$\begin{pmatrix} \blacksquare & \blacksquare & \blacksquare \\ 0 & \blacksquare & \blacksquare \\ 0 & 0 & \blacksquare \end{pmatrix}$$

lower triangular matrix: $A \in \mathbb{R}^{n \times n}$

$a_{ij} = 0$ for $i < j$

$$\begin{pmatrix} \blacksquare & 0 & 0 & 0 \\ \blacksquare & \blacksquare & 0 & 0 \\ \blacksquare & \blacksquare & \blacksquare & 0 \\ \blacksquare & \blacksquare & \blacksquare & \blacksquare \end{pmatrix}$$

symmetric matrix: $A \in \mathbb{R}^{n \times n}$

$$\begin{pmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{pmatrix} \qquad \begin{pmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \\ 3 & 2 & -5 \end{pmatrix}$$

$a_{ij} = a_{ji}$ for all i,j

skew-symmetric matrix: $A \in \mathbb{R}^{n \times n}$

$$\begin{pmatrix} \blacksquare & \blacksquare & \blacksquare \\ -\blacksquare & \blacksquare & \blacksquare \\ -\blacksquare & -\blacksquare & \blacksquare \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{pmatrix}$$

$a_{ij} = -a_{ji}$ for all i,j