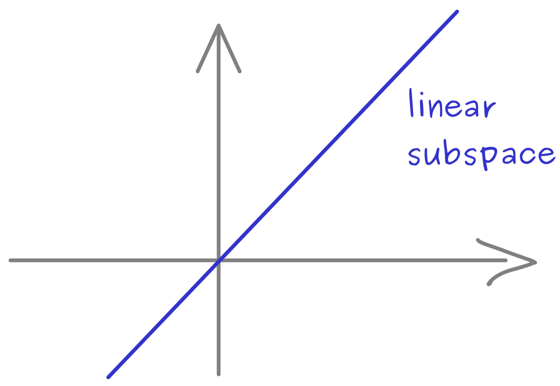
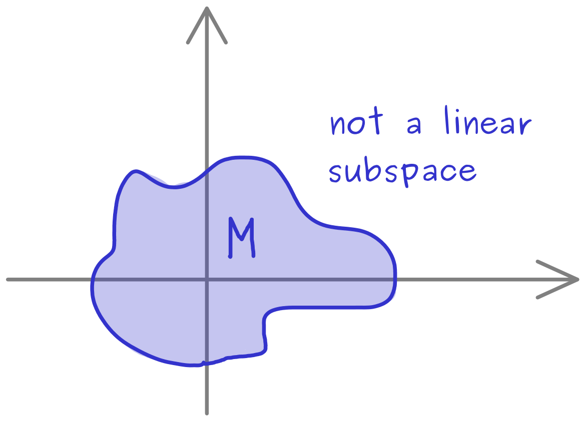
Linear Algebra – Part 8

linear span/ linear hull/ span





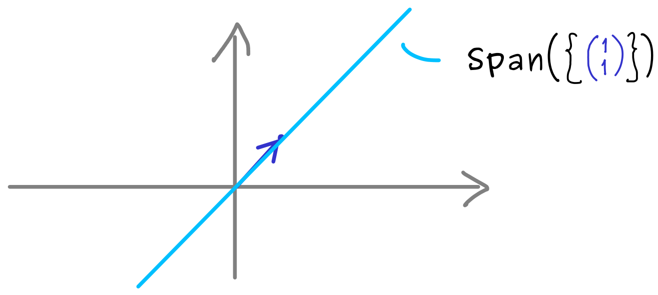
$\mathrm{Span}(M)$
- linear subspace
- contains all linear combinations of vectors from M
- smallest subspace with this property

Definition: $M \subseteq \mathbb{R}^n$ non-empty

$$\mathrm{Span}(M) := \left\{ u \in \mathbb{R}^n \;\middle|\; \text{there are } \lambda_j \in \mathbb{R} \text{ and } u^{(j)} \in M \text{ with: } u = \sum_{j=1}^{k} \lambda_j u^{(j)} \right\}$$

$$\mathrm{Span}(\emptyset) := \{0\}$$

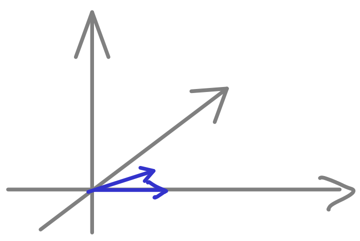
Example: (a) $\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\} \subseteq \mathbb{R}^2$



$$\mathrm{Span}\left(\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}\right) := \left\{ u \in \mathbb{R}^n \;\middle|\; \text{there is } \lambda \in \mathbb{R} \text{ such that } u = \lambda \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$$

$$\mathrm{Span}\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = \left\{ \lambda \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \;\middle|\; \lambda \in \mathbb{R} \right\} = \mathbb{R} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

(b) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\} \subseteq \mathbb{R}^3$



$$\mathrm{Span}\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right) = \left\{ \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \;\middle|\; x, y \in \mathbb{R} \right\}$$

We say: the subspace is generated by the vectors $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

Example: $\mathbb{R}^n = \mathrm{Span}(e_1, e_2, \dots, e_n)$