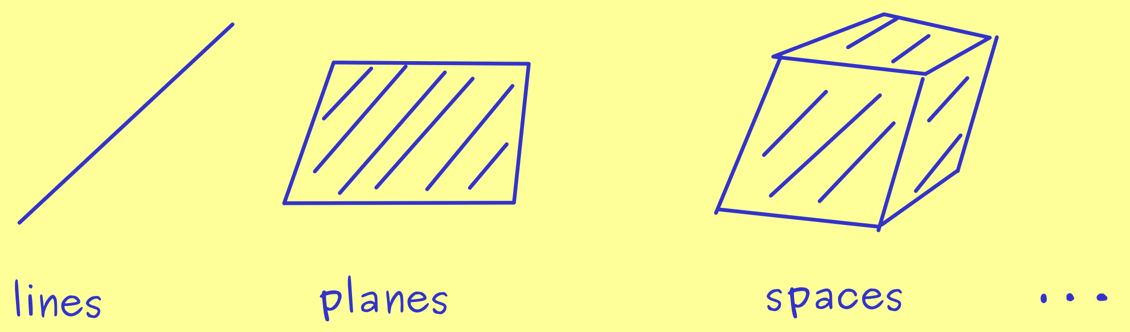
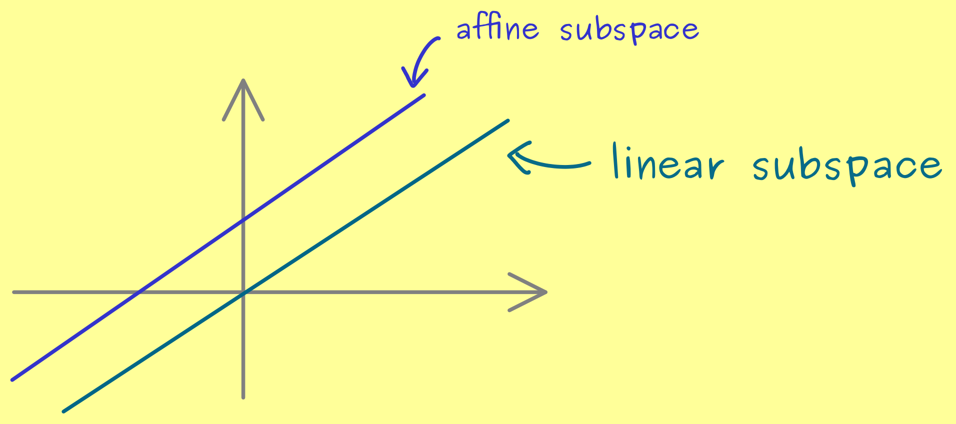
Linear Algebra – Part 6

(linear) subspaces:



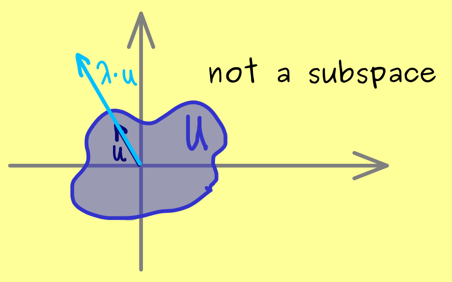
with special properties

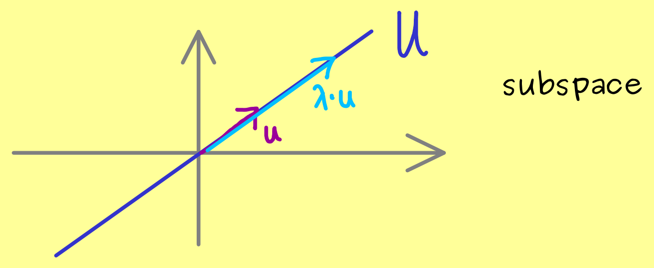
In $\mathbb{R}^2$:



Definition: $U \subseteq \mathbb{R}^n$, $U \neq \emptyset$, is called a (linear) subspace of $\mathbb{R}^n$ if all linear combinations in U remain in U:

$$\begin{matrix} u^{(1)}, u^{(2)}, \dots, u^{(k)} \in U \\ \lambda_1, \lambda_2, \dots, \lambda_k \in \mathbb{R} \end{matrix} \implies \sum_{j=1}^{k} \lambda_j u^{(j)} \in U$$





Characterisation for subspaces:

$U \subseteq \mathbb{R}^n$ is a subspace $\iff$

(a) $0 \in U$

(b) $u \in U$, $\lambda \in \mathbb{R}$ $\implies$ $\lambda \cdot u \in U$

(c) $u, v \in U$ $\implies$ $u + v \in U$

Examples: $U = \{0\}$ subspace!

$U = \mathbb{R}^n$

all other subspaces U satisfy: $\{0\} \subsetneqq U \subsetneqq \mathbb{R}^n$