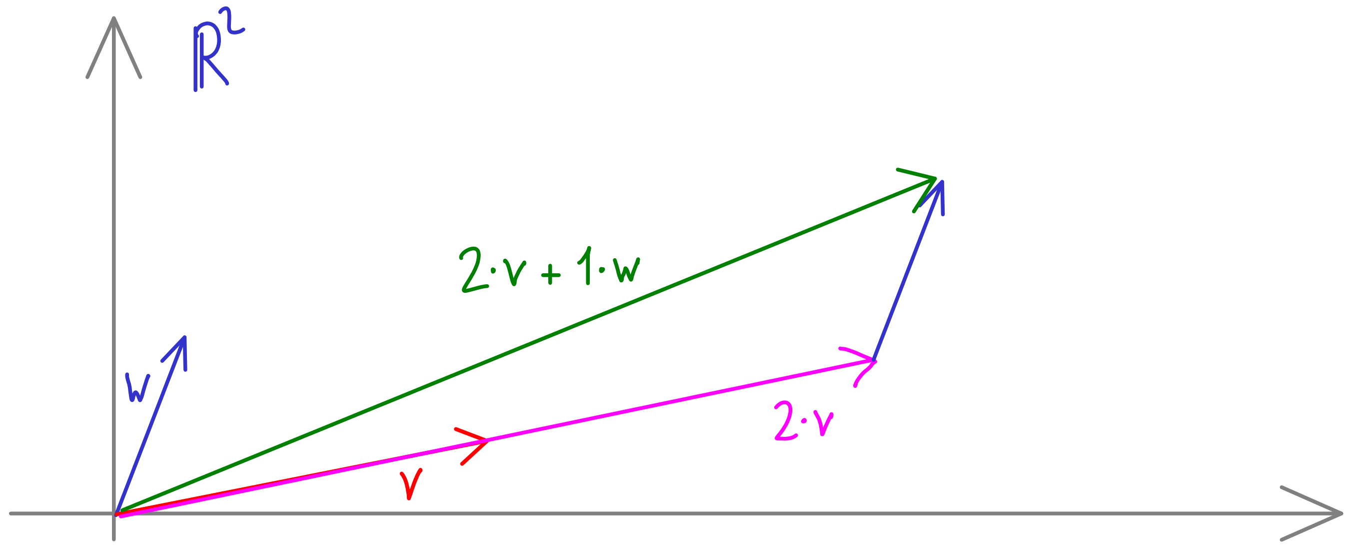
Linear Algebra – Part 3

$\mathbb{R}^2$ with two operations $(\cdot, +)$ is a vector space.

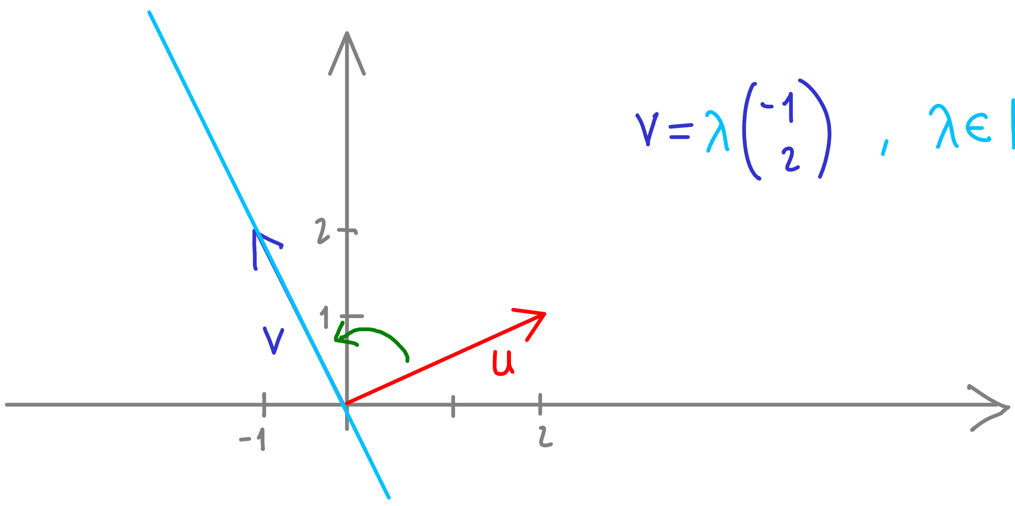
combine them: linear combination



Definition: For vectors $v^{(1)}, v^{(2)}, \dots, v^{(k)} \in \mathbb{R}^2$ and scalars $\lambda_1, \lambda_2, \dots, \lambda_k \in \mathbb{R}$,

the vector $$V = \sum_{j=1}^{k} \lambda_j v^{(j)}$$ is called a linear combination.

Question: Which vectors $v \in \mathbb{R}^2$ are perpendicular to the vector $u = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$?



$v = \lambda \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \quad \lambda \in \mathbb{R}$

Answer: $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ and $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ are orthogonal

$$\iff \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda \cdot \begin{pmatrix} -u_2 \\ u_1 \end{pmatrix} \quad \text{for some } \lambda \in \mathbb{R}$$

$$\iff u_1 \cdot v_1 = -\underbrace{u_1 \lambda}_{v_2} u_2 \quad \text{and} \quad u_2 v_2 = \underbrace{u_2 \cdot \lambda}_{-v_1} \cdot u_1 \quad \text{for some } \lambda \in \mathbb{R}$$

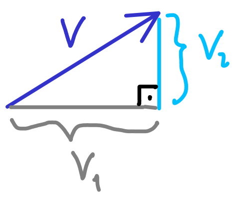
$$\iff u_1 \cdot v_1 = -v_2 \cdot u_2 \quad \text{and} \quad u_2 v_2 = -v_1 \cdot u_1$$

$$\iff \underbrace{u_1 v_1 + u_2 v_2}_{=:\ \langle u, v \rangle} = 0$$

$\langle u, v \rangle$ (standard) inner product

↳ more structure (geometry)

Definition:



length of $v = \sqrt{v_1^2 + v_2^2}$

$\|v\| := \sqrt{\langle v, v \rangle}$ is called the (standard) norm

Euclidean
//
(standard)