BECOME A MEMBER

ON STEADY

The Bright Side of
Mathematics

Linear Algebra – Part 47

Leibniz formula:

$$\det\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} = \sum_{(j_1,\dots,j_n)\in S_n} \operatorname{sgn}((j_1,\dots,j_n))\, a_{j_1,1}\, a_{j_2,2} \cdots a_{j_n,n}$$

how many terms?

For $n=2$: $(1,2)$, $(2,1)$ — 2 permutations

For $n=3$: $(1,2,3)$, $(2,3,1)$, $(3,1,2)$, $(1,3,2)$, $(3,2,1)$, $(2,1,3)$ — 6 permutations (rule of Sarrus)

For $n=4$: ... — 24 permutations

For n: ... — $n!$ permutations

Rule of Sarrus:

$$\det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = + + + - - -$$

Example:

$$\det\begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & -2 \\ 1 & 4 & 1 \end{pmatrix} = -1 + 8 + (-4) - (-1) - (-8) - 4 = 8$$