BECOME A MEMBER

ON STEADY

The Bright Side of Mathematics

Linear Algebra – Part 36

System of linear equations:

$$2x_1 + 3x_2 + 4x_3 = 1$$
$$4x_1 + 6x_2 + 9x_3 = 1$$
$$2x_1 + 4x_2 + 6x_3 = 1$$

3 equations
3 unknowns

Short notation: $Ax = b$ $\xrightarrow{\text{augmented matrix}}$ $(A \mid b)$

$$\left(\begin{array}{ccc|c} 2 & 3 & 4 & 1 \\ 4 & 6 & 9 & 1 \\ 2 & 4 & 6 & 1 \end{array}\right)$$

Example:

$x_1 + 3x_2 = 7$ (equation 1)

$2x_1 - x_2 = 0$ (equation 2) $\rightsquigarrow$ $x_2 = 2x_1$

put in equation 1

$\Rightarrow x_1 + 3(2x_1) = 7$

$\Leftrightarrow 7x_1 = 7 \Leftrightarrow x_1 = 1 \rightsquigarrow x_2 = 2$

$\Rightarrow$ Only possible solution: $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ Check? ✓

$\Rightarrow$ The system has a unique solution given by $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Better method: Gaussian elimination

Example:

$x_1 + 3x_2 = 7$ (equation 1)

$2x_1 - x_2 = 0$ (equation 2) $- 2 \cdot$(equation 1)

eliminate x_1

$\rightsquigarrow$

$x_1 + 3x_2 = 7$ (equation 1)

$0 - 7x_2 = -14$ (equation 2)$' \cdot (-\frac{1}{7})$

$\rightsquigarrow$

$x_1 + 3x_2 = 7$ (equation 1)

$x_2 = 2$ (equation 2)''

$\Rightarrow x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ solution