Linear Algebra – Part 32

Transposition: changing the roles of columns and rows

$$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}^T = \begin{pmatrix} a_1 & a_2 & \cdots & a_n \end{pmatrix}$$

$$\begin{pmatrix} a_1 & a_2 & \cdots & a_n \end{pmatrix}^T = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

For $a \in \mathbb{R}^n$ we have: $(a^T)^T = a$

Definition: For $A \in \mathbb{R}^{m \times n}$ we define $A^T \in \mathbb{R}^{n \times m}$ (transpose of A) by:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}$$

Examples:

(a) $$A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 1 & 2 \\ 2 & 0 \\ 0 & 3 \\ 1 & 0 \end{pmatrix}$$

(b) $$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

(c) $$A = \begin{pmatrix} 1 & 4 & 5 \\ 4 & 2 & 0 \\ 5 & 0 & 3 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 1 & 4 & 5 \\ 4 & 2 & 0 \\ 5 & 0 & 3 \end{pmatrix}$$ (symmetric matrix)

Remember: $(AB)^T = B^T A^T$