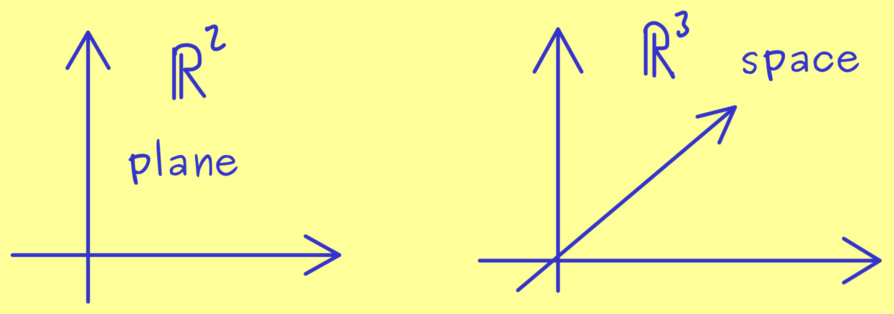
The Bright Side of
Mathematics

Linear Algebra – Part 5



$$\mathbb{R}^n = \underbrace{\mathbb{R} \times \cdots \times \mathbb{R}}_{n \text{ times}} \quad \text{for} \quad n \in \mathbb{N}$$

write $v \in \mathbb{R}^n$ in column form: $v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \in \mathbb{R}^n$

addition: $u + v = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} + \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} := \begin{pmatrix} u_1 + v_1 \\ \vdots \\ u_n + v_n \end{pmatrix}$

scalar multiplication: $\lambda \cdot u = \lambda \cdot \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} := \begin{pmatrix} \lambda \cdot u_1 \\ \vdots \\ \lambda \cdot u_n \end{pmatrix}$

$\hookrightarrow$ $(\mathbb{R}^n, +, \cdot)$ is a vector space

Properties: (a) $(\mathbb{R}^n, +)$ is an abelian group:

(1) $u + (v + w) = (u + v) + w$ (associativity of $+$)

(2) $v + 0 = v$ with $0 = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$ (neutral element)

(3) $v + (-v) = 0$ with $-v = \begin{pmatrix} -v_1 \\ \vdots \\ -v_n \end{pmatrix}$ (inverse elements)

(4) $v + w = w + v$ (commutativity of $+$)

(b) scalar multiplication is compatible: $\cdot : \mathbb{R} \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$

(5) $\lambda \cdot (\mu \cdot v) = (\lambda \cdot \mu) \cdot v$

(6) $1 \cdot v = v$

(c) distributive laws:

(7) $\lambda \cdot (v + w) = \lambda \cdot v + \lambda \cdot w$

(8) $(\lambda + \mu) \cdot v = \lambda \cdot v + \mu \cdot v$

Canonical unit vectors: $e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \dots, \quad e_n = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$

$v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \in \mathbb{R}^n$ can be written as a linear combination: $v = \sum_{j=1}^{n} v_j \cdot e_j$