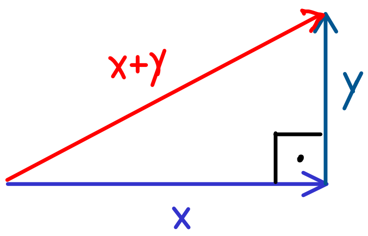
Hilbert Spaces - Part 7



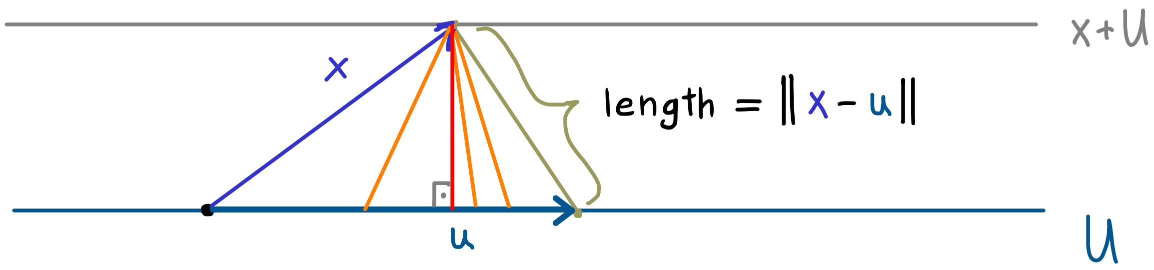
choose x, y orthogonal: $\langle x, y \rangle = 0$

Pythagorean theorem: $(X, \langle \cdot , \cdot \rangle)$ inner product space with induced norm $\| \cdot \|$.

For any $x, y \in X$ with $x \perp y$, we have:

$$\| x+y \|^2 = \langle x+y , x+y \rangle = \langle x, x \rangle + \underbrace{\langle y, x \rangle}_{=0} + \underbrace{\langle x, y \rangle}_{=0} + \langle y, y \rangle$$
$$= \| x \|^2 + \| y \|^2$$

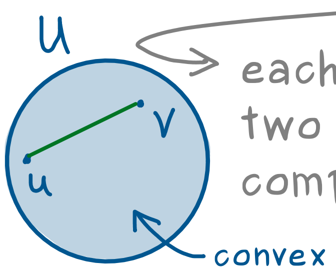
Approximation Formula

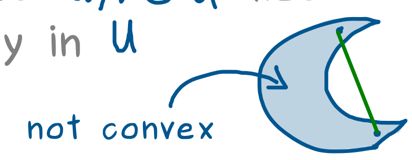


distance between $x+U$ and U : $\inf \{ \| x - u \| \mid u \in U \} =: \mathrm{dist}(x, U)$

Theorem: Let $(X, \langle \cdot , \cdot \rangle)$ be a Hilbert space, $U \subseteq X$ be closed and convex.

each connection line between two points $u, v \in U$ lies completely in U





For every $x \in X$ there exists a unique best approximation:

$$x|_U \in U$$

This means: $\| x - x|_U \| = \mathrm{dist}(x, U)$