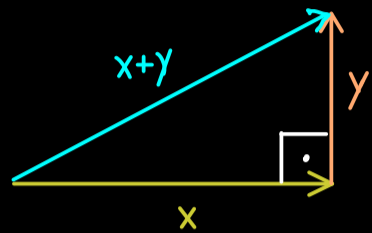


## Hilbert Spaces - Part 7



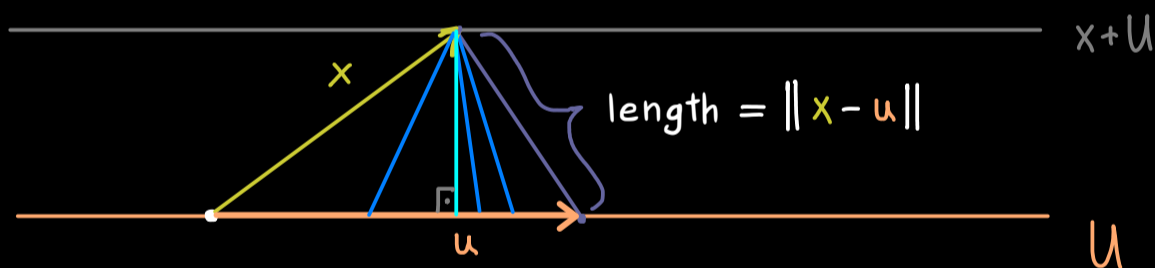
choose  $x, y$  orthogonal:  $\langle x, y \rangle = 0$

Pythagorean theorem:  $(X, \langle \cdot, \cdot \rangle)$  inner product space with induced norm  $\|\cdot\|$ .

For any  $x, y \in X$  with  $x \perp y$ , we have:

$$\begin{aligned} \|x+y\|^2 &= \langle x+y, x+y \rangle = \langle x, x \rangle + \overbrace{\langle y, x \rangle}^{=0} + \overbrace{\langle x, y \rangle}^{=0} + \langle y, y \rangle \\ &= \|x\|^2 + \|y\|^2 \end{aligned}$$

### Approximation Formula



distance between  $x+U$  and  $U$ :  $\inf \{ \|x-u\| \mid u \in U \} =: \text{dist}(x, U)$

Theorem: Let  $(X, \langle \cdot, \cdot \rangle)$  be a Hilbert space,  $U \subseteq X$  be closed and convex.

For every  $x \in X$  there exists a unique best approximation:

$$x|_U \in U$$

This means:  $\|x - x|_U\| = \text{dist}(x, U)$

