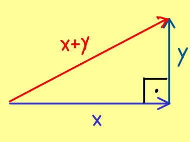
BECOME A MEMBER

ON STEADY

The Bright Side of Mathematics

Hilbert Spaces - Part 7



choose x, y orthogonal: $\langle x, y \rangle = 0$

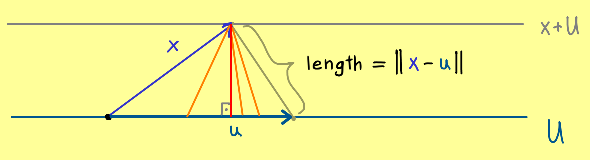
Pythagorean theorem: $(X, \langle \cdot, \cdot \rangle)$ inner product space with induced norm $\|\cdot\|$.

For any $x, y \in X$ with $x \perp y$, we have:

$$\|x+y\|^2 = \langle x+y, x+y \rangle = \langle x, x \rangle + \underbrace{\langle y, x \rangle}_{=0} + \underbrace{\langle x, y \rangle}_{=0} + \langle y, y \rangle$$

$$= \|x\|^2 + \|y\|^2$$

Approximation Formula

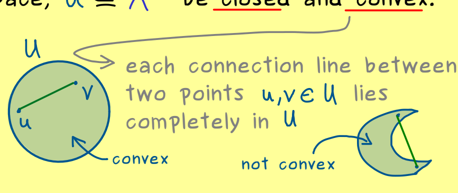


distance between $x+U$ and U : $\inf \{ \|x-u\| \mid u \in U \} =: \text{dist}(x, U)$

Theorem: Let $(X, \langle \cdot, \cdot \rangle)$ be a Hilbert space, $U \subseteq X$ be closed and convex.

For every $x \in X$ there exists a unique best approximation:

$x|_U \in U$



This means: $\|x - x|_U\| = \text{dist}(x, U)$