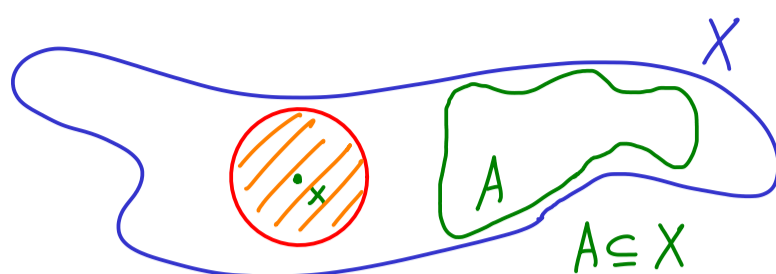


Functional analysis - part 3

(X, d) metric space



$$B_\epsilon(x) := \{ y \in X \mid d(x, y) < \epsilon \} \quad (\text{open ball of radius } \epsilon > 0 \text{ centered at } x)$$

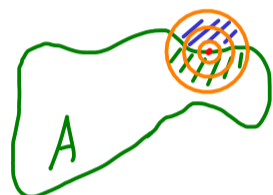
Notions:

(1) Open sets:



$A \subseteq X$ is called open if for each $x \in A$ there is an open ball with $B_\epsilon(x) \subseteq A$.

(2) Boundary points:



$A \subseteq X$. $x \in X$ is called a boundary point for A if for all $\epsilon > 0$: $B_\epsilon(x) \cap A \neq \emptyset$ and $B_\epsilon(x) \cap A^c \neq \emptyset$ [$A^c := X \setminus A$]

$$\partial A := \{ x \in X \mid x \text{ is boundary point for } A \}$$

Remember: A open $\Leftrightarrow A \cap \partial A = \emptyset$

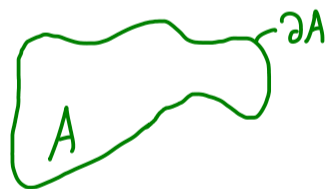
(3) Closed sets:



$A \subseteq X$ is called closed if $A^c := X \setminus A$ is open.

Remember: A closed $\Leftrightarrow A \cup \partial A = A$

(4) Closure:



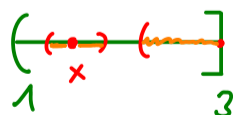
$$\bar{A} := A \cup \partial A \quad (\text{always closed!})$$

Example:

$X := (1, 3] \cup (4, \infty)$, $d(x, y) := |x - y|$, (X, d) is a metric space

(a)

$A := (1, 3] \subseteq X$ open?



For $x \in A$, $x \neq 3$, define $\epsilon := \frac{1}{2} \min(|1-x|, |3-x|)$. Then $B_\epsilon(x) \subseteq A$.
For $x = 3$: $B_1(x) = \{ y \in X \mid d(x, y) < 1 \} = (2, 3] \subseteq A$ } $\Rightarrow A$ is open

(b) A is also closed!

(c) $C := (1, 2]$, $\partial C = \{2\}$, $\bar{C} = C$

