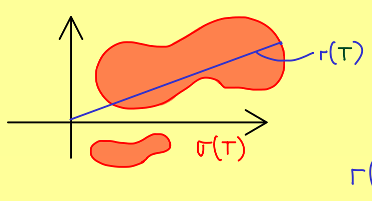
The Bright Side of
Mathematics

Functional analysis – part 32



$T: X \to X$ bounded linear operator

$r(T) \leq \|T\|$

For normal operators: $r(T) = \|T\|$

X is a complex Hilbert space

Definition: Let X be a Hilbert space and $T: X \to X$ a bounded linear operator. The bounded linear operator $T^*: X \to X$ defined by

$$\langle y, Tx \rangle = \langle T^* y, x \rangle \quad \text{for all } x, y \in X$$

is called the adjoint operator of T.

Definition: Let X be a Hilbert space and $T: X \to X$ a bounded linear operator. T is called
(1) self-adjoint if $T^* = T$
(2) skew-adjoint if $T^* = -T$
(3) normal if $T^*T = TT^*$

Proposition: T normal $\Rightarrow$ $r(T) = \|T\|$