BECOME A MEMBER

ON STEADY

The Bright Side of Mathematics

Functional Analysis – Part 34

Spectral theorem of compact operators

Let X be a complex Hilbert space and $T: X \to X$ be a compact operator.
Assume that T is self-adjoint $(T^* = T)$ or normal $(T^*T = TT^*)$.

Then there is an orthonormal system $\{e_i \mid i \in I\}$ with $I \subseteq \mathbb{N}$
and a sequence $(\lambda_i)_{i \in I}$ in $\mathbb{C} \setminus \{0\}$ with $\lambda_i \xrightarrow{i \to \infty} 0$ (if I infinite)
such that:

$$X = \mathrm{Ker}(T) \oplus^{\perp} \overline{\mathrm{Span}(e_i \mid i \in I)}$$

orthogonal sum: $X = U \oplus^{\perp} V$ means:
for each $x \in U$ there is $u \in U$, $v \in V$:
- $x = u + v$
- $u \perp v$

unique!

and $$Tx = \sum_{k \in I} \lambda_k e_k \langle e_k, x \rangle \quad \text{for } x \in X$$

eigenvector to λ_k

eigenvalue

and $$\|T\| = \sup_{k \in I} |\lambda_k| .$$