

**Exercise 1. Equivalent norm and completeness**

Let  $X$  be a vector space. Two norms on  $X$ , denoted by  $\|\cdot\|_1$  and  $\|\cdot\|_2$ , are called **equivalent** if there are two constants  $c, C > 0$  such that  $c\|x\|_1 \leq \|x\|_2 \leq C\|x\|_1$  holds for all  $x \in X$ . Show the following:

(a)  $U$  is open in  $(X, \|\cdot\|_1)$  if and only if  $U$  is open in  $(X, \|\cdot\|_2)$ .

(b)  $(X, \|\cdot\|_1)$  is a Banach space if and only if  $(X, \|\cdot\|_2)$  is a Banach space.