

**Exercise 1. Absolutely convergent series**

Let  $(X, \|\cdot\|)$  be a normed space. Show the equivalence of the following statements:

- (a)  $(X, \|\cdot\|)$  is a Banach space
- (b) Every absolutely convergent series is convergent, which means for every sequence  $(x_n) \subseteq X$  with  $\sum_{n=1}^{\infty} \|x_n\| < \infty$ , there is an element  $x \in X$  such that

$$\left\| x - \sum_{n=1}^N x_n \right\| \xrightarrow{N \rightarrow \infty} 0$$