

Definition 1. Accumulation Value

Let (X, d) be a metric space and (x_n) a sequence in it.

- (1) A point $a \in X$ is called an **accumulation value** of (x_n) if there is a subsequence $(x_{n_k})_{k \in \mathbb{N}}$ that converges to a .
- (2) The sequence is called **bounded** if there is a ball $B_R(x_0)$ such that $x_n \in B_R(x_0)$ for all $n \in \mathbb{N}$.

Exercise 2. Cauchy sequences

Let (X, d) be a metric space. Which of the following claims are correct and why?

- (a) Each Cauchy sequence that has an accumulation value is a convergent sequence
- (b) Each bounded sequence contains at least one accumulation value.
- (c) Each bounded sequence is convergent.
- (d) Each convergent sequence is a Cauchy sequence.
- (e) Each convergent sequence is a bounded sequence.
- (f) Each Cauchy sequence is bounded.