

**Exercise 1. Different Metrics**

Show that we have well-defined metrics in the following cases:

(a)  $M_1 := \{0, 1\}^n$ ,  $n \in \mathbb{N}$  and

$$d_1: M_1 \times M_1 \rightarrow \mathbb{R}, d_1(x, y) := |\{i \in \{1, \dots, n\} : x_i \neq y_i\}|$$

(b)  $M_2 := \mathbb{C}$  and

$$d_2: M_2 \times M_2 \rightarrow \mathbb{R}, d_2(z, w) := \begin{cases} |z - w| & \text{if } w = \lambda z \text{ for some } \lambda \in \mathbb{R} \setminus \{0\} \\ |z| + |w| & \text{else} \end{cases}$$

(c)  $M_3 := \mathbb{C}$  and

$$d_3: M_3 \times M_3 \rightarrow \mathbb{R}, d_3(x, y) := \frac{|x - y|}{1 + |x - y|}$$