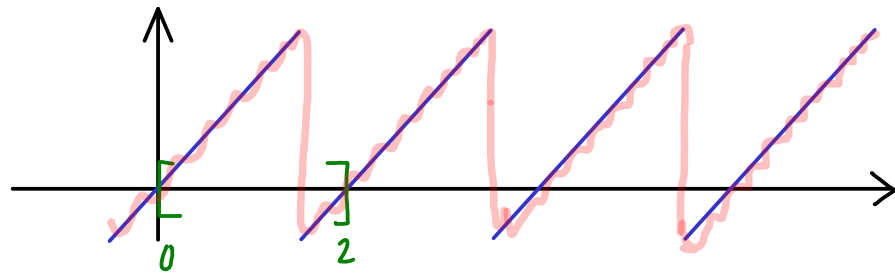
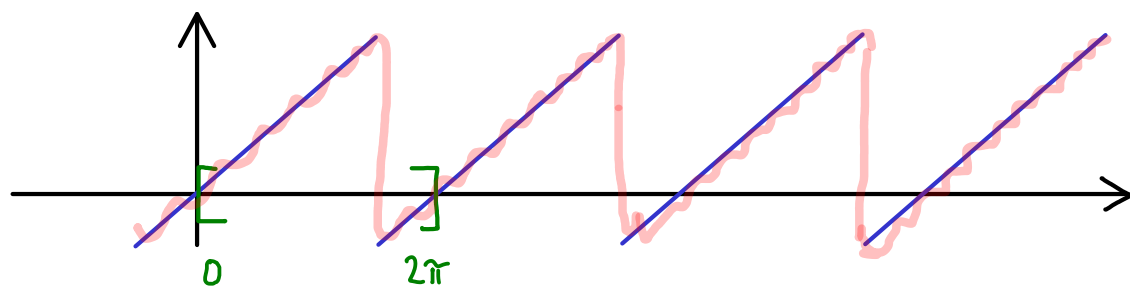


Fourier Transform - Part 2

Idea of Fourier series:



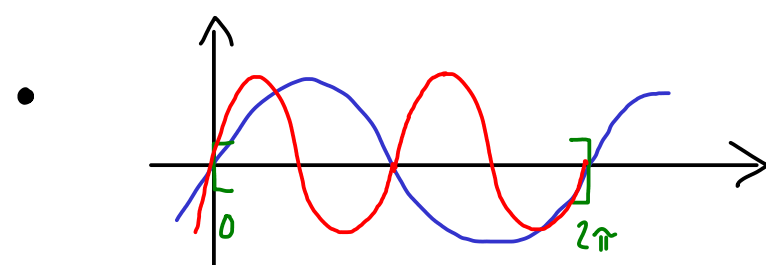
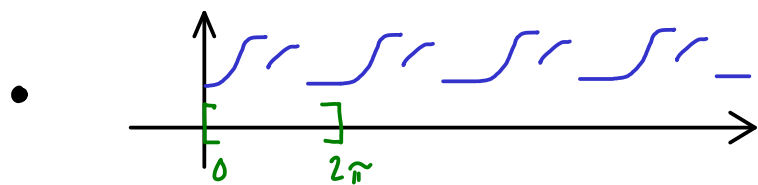
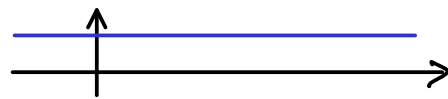
The function is 2-periodic: $f(x+2) = f(x)$ for all $x \in \mathbb{R}$



$$\mathcal{F}_{2\pi\text{-per}}(\mathbb{R}, \mathbb{R}) = \{ f: \mathbb{R} \rightarrow \mathbb{R} \mid f(x+2\pi) = f(x) \text{ for all } x \in \mathbb{R} \}$$

↳ real vector space

Example: • constant function $f(x) = 5$



$$x \mapsto \sin(x)$$

$$x \mapsto \sin(2x)$$

Proposition: $U \subseteq \mathcal{F}_{2\pi\text{-per}}(\mathbb{R}, \mathbb{R})$ given by

$$U := \left\{ \begin{array}{l} x \mapsto \sin(x), x \mapsto \sin(2x), x \mapsto \sin(3x), \dots, \\ x \mapsto 1, x \mapsto \cos(x), x \mapsto \cos(2x), x \mapsto \cos(3x), \dots \end{array} \right\}$$

odd functions
↙
↘
even functions

is linearly independent.

Definition: A linear combination $f \in \text{Span}(U)$, $f: \mathbb{R} \rightarrow \mathbb{R}$, is called
(real) trigonometric polynomial:

$$f(x) = a_0 + \sum_{k=1}^n a_k \cdot \cos(k \cdot x) + \sum_{k=1}^n b_k \cdot \sin(k \cdot x), \quad a_i, b_i \in \mathbb{R}$$

For $\mathcal{F}_{2\pi\text{-per}}(\mathbb{R}, \mathbb{C})$, we have a (complex) trigonometric polynomial:

$$f(x) = \sum_{k=-n}^n c_k \cdot \exp(i \cdot k \cdot x), \quad c_k \in \mathbb{C}$$