Integration Exercises

Exercise 1. Determine the antiderivatives of cosh and sinh.

primitive

$$\int \cosh(x)\,dx = \int \frac{e^x + e^{-x}}{2}\,dx = \frac{1}{2}\left(\int e^x\,dx + \int e^{-x}\,dx\right) = \frac{1}{2}\left(e^x + c_1 + (-e^{-x}) + c_2\right) =$$

$$= \underbrace{\frac{e^x - e^{-x}}{2}}_{\sinh(x)} + \underbrace{\frac{c_1 + c_2}{2}}_{c_3 \in \mathbb{R}} = \sinh(x) + c_3 \qquad c_3 \in \mathbb{R}$$

$(e^x)' = e^x \qquad (e^{-x})' = e^{-x}(-1) \qquad (-e^{-x})' = e^{-x} \qquad c_1, c_2 \in \mathbb{R}$

$$\int \sinh(x)\,dx = \int \frac{e^x - e^{-x}}{2}\,dx = \frac{1}{2}\left(\int e^x\,dx - \int e^{-x}\,dx\right) = \frac{1}{2}\left(e^x + e^{-x} + c\right) = \underbrace{\frac{e^x + e^{-x}}{2}}_{\cosh(x)} + \underbrace{\tilde{c}}_{\in \mathbb{R}} =$$

$$= \cosh(x) + c, \quad c \in \mathbb{R}$$