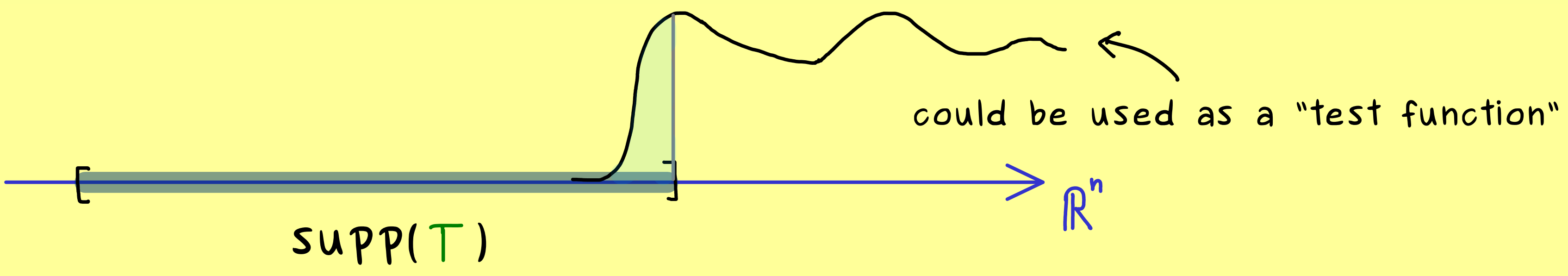
BECOME A MEMBER

ON STEADY

The Bright Side of
Mathematics

Distributions - Part 16

$T \in \mathcal{D}'(\mathbb{R}^n) \rightsquigarrow \operatorname{supp}(T) \subseteq \mathbb{R}^n$ closed set

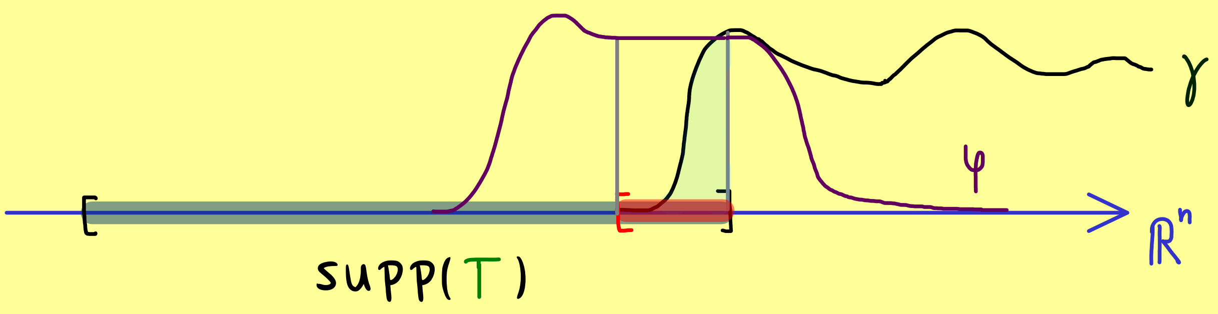


<u>Definition:</u> For $T \in \mathcal{D}'(\mathbb{R}^n)$, we define: $\mathcal{E}_T := \left\{ \gamma \in C^\infty(\mathbb{R}^n) \;\middle|\; \begin{array}{l}\operatorname{supp}(T) \cap \operatorname{supp}(\gamma) \\ \text{is compact in } \mathbb{R}^n\end{array} \right\}$

Extension for T to $\mathcal{E}_T$:

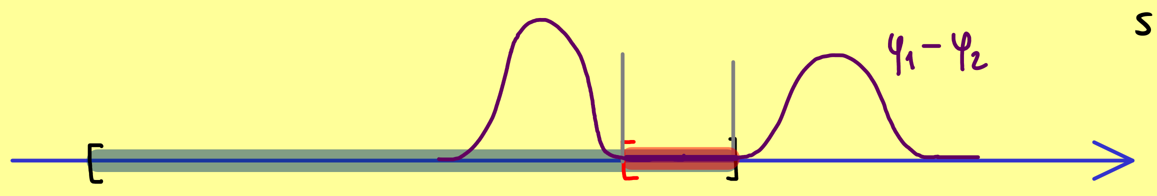
$$T(\gamma) = \langle T, \gamma \rangle := \langle T, \varphi \cdot \gamma \rangle \quad \text{for } \varphi \in \mathcal{D}(\mathbb{R}^n) \text{ with}$$

$\varphi(x) = 1$, for all x in an open set that contains: $\operatorname{supp}(T) \cap \operatorname{supp}(\gamma)$



$\operatorname{supp}(\varphi \cdot \gamma) = \operatorname{supp}(\varphi) \cap \operatorname{supp}(\gamma)$

<u>Is it well-defined?</u> $\varphi_1, \varphi_2 \in \mathcal{D}(\mathbb{R}^n)$, $\varphi_1(x) = \varphi_2(x) = 1$, for all x in an open set that contains: $\operatorname{supp}(T) \cap \operatorname{supp}(\gamma)$



$$\langle T, \varphi_1 \gamma \rangle - \langle T, \varphi_2 \gamma \rangle = \langle T, (\varphi_1 - \varphi_2)\gamma \rangle = 0$$

<u>Properties:</u> $T : \mathcal{E}_T \longrightarrow \mathbb{R}$ (or $\mathbb{C}$) satisfies:

(1) It's a linear functional on $\mathcal{E}_T$.

(2) Distributional derivatives: $\langle D^\alpha T, \gamma \rangle = (-1)^{|\alpha|} \langle T, D^\alpha \gamma \rangle$

<u>Common notation:</u> For a distribution T where $\operatorname{supp}(T)$ is compact in $\mathbb{R}^n$:

$$T \in \mathcal{E}'(\mathbb{R}^n)$$