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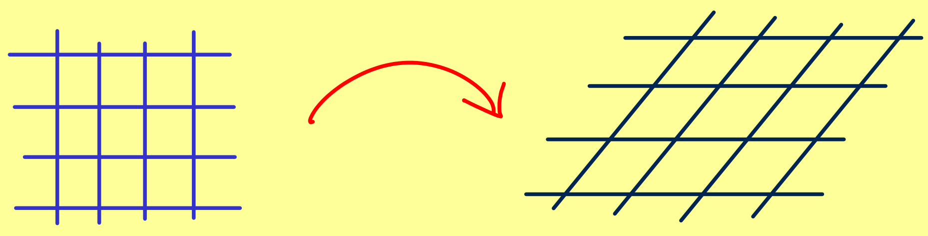
ON STEADY

The Bright Side of Mathematics

Distributions – Part 9

invertible linear map $A: \mathbb{R}^n \to \mathbb{R}^n$



$$f \in \mathcal{L}^1_{loc}(\mathbb{R}^n) \quad \Rightarrow \quad f \circ A \in \mathcal{L}^1_{loc}(\mathbb{R}^n)$$

$$\langle T_{f\circ A}, \varphi \rangle = \int_{\mathbb{R}^n} f(Ax)\varphi(x)\,dx = \frac{1}{|\det(A)|}\int_{\mathbb{R}^n} f(\underbrace{Ax}_{y})\varphi(x)\,\underbrace{|\det(A)|\,dx}_{dy}$$

$$= \frac{1}{|\det(A)|}\int_{\mathbb{R}^n} f(y)\varphi(A^{-1}y)\,dy = \left\langle T_f, \frac{1}{|\det(A)|}\varphi\circ A^{-1} \right\rangle$$

Definition: Let $T \in \mathcal{D}'(\mathbb{R}^n)$ and $A: \mathbb{R}^n \to \mathbb{R}^n$ be an invertible linear map.

Define: $\langle T \circ A, \varphi \rangle := \left\langle T, \frac{1}{|\det(A)|}\varphi \circ A^{-1} \right\rangle$

Strange notation: $\delta(x)$ denotes the delta distribution

Or: (with strange notation) $\langle T(Ax), \varphi(x) \rangle := \left\langle T(x), \frac{1}{|\det(A)|}\varphi(A^{-1}x) \right\rangle$

For $b \in \mathbb{R}^n$ define: $\langle T(Ax+b), \varphi(x) \rangle := \left\langle T(x), \frac{1}{|\det(A)|}\varphi(A^{-1}(x-b)) \right\rangle$

and: $G^{-1} \in C^\infty(\mathbb{R}^n)$

For $G \in C^\infty(\mathbb{R}^n)$ bijective, define: $\langle T(Gx), \varphi(x) \rangle := \left\langle T(x), \frac{1}{|\det(\underbrace{J_G(x)}_{\text{Jacobian matrix of } G})|}\varphi(G^{-1}x) \right\rangle$

Example: $A: \mathbb{R}^n \to \mathbb{R}^n$ rotation $\left(A^{-1} = A^T,\ |\det(A)| = 1\right)$

$$\langle \delta(Ax), \varphi(x) \rangle = \langle \delta(x), \varphi(A^{-1}x) \rangle = \varphi(A^{-1}0) = \varphi(0)$$

$$= \langle \delta(x), \varphi(x) \rangle \Rightarrow \delta(Ax) = \delta(x)$$

$\Rightarrow$ delta distribution is rotational invariant