## Complex Analysis - Part 12

$$\int (z) = \sum_{k=0}^{\infty} a_k \cdot z^k$$

holomorphic on its open disc of convergence

$$f(z) = \sum_{k=0}^{\infty} a_k \cdot z^k$$
holomorphic on its open disc of constant  $z$ 

$$f' \text{ exists and is a power series}$$

$$f'' \text{ exists and is a power series}$$

$$\vdots$$

Examples: (1) 
$$exp(\xi) := \sum_{k=0}^{\infty} \frac{\xi^k}{k!}$$
 (radius of convergence:  $\Gamma = \infty$ ) 
$$exp(\xi) = \sum_{k=1}^{\infty} \frac{k \cdot \xi^{k-1}}{k!} = \sum_{k=1}^{\infty} \frac{\xi^{k-1}}{(k-1)!} = \sum_{m=0}^{\infty} \frac{\xi^m}{m!} = exp(\xi)$$

(2) 
$$\cos(z) := \sum_{k=0}^{\infty} (-1)^k \frac{z^{2\cdot k}}{(2\cdot k)!}$$

$$= \begin{cases} z^k, & k = 0, 4, 8, \dots \\ iz^k, & k = 1, 5, 9, 13, \dots \\ -z^k, & k = 2, 6, 10, \dots \\ -iz^k, & k = 3, 7, 11, \dots \end{cases}$$
connection?  $e \times p(i \cdot z) = \sum_{k=0}^{\infty} \frac{(iz)^k}{k!} = \begin{cases} -iz^k, & k = 3, 7, 11, \dots \\ -iz^k, & k = 3, 7, 11, \dots \end{cases}$ 

$$e \times p(-i \cdot z) = \sum_{k=0}^{\infty} \frac{(-iz)^k}{k!} > \begin{cases} z^k, & k = 0, 4, 8, \dots \\ -iz^k, & k = 1, 5, 9, 13, \dots \\ -z^k, & k = 2, 6, 10, \dots \\ +iz^k, & k = 3, 7, 11, \dots \end{cases}$$

$$e \times p(i \cdot z) + e \times p(-i \cdot z) = \sum_{m=0}^{\infty} (-1)^m \frac{z^{2m}}{(2m)!} \cdot 2 = 2 \cdot cos(z)$$

$$\implies cos(z) = \frac{1}{2} \left( e \times p(i \cdot z) + e \times p(-i \cdot z) \right)$$

$$\implies cos^1(z) = \frac{i}{2} \left( e \times p(i \cdot z) - e \times p(-i \cdot z) \right) = -sin(z)$$