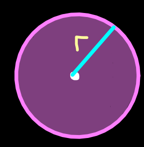
Complex Analysis – Part 12



$$f(z) = \sum_{k=0}^{\infty} a_k \cdot z^k$$

holomorphic on its open disc of convergence

f' exists and is a power series

f'' exists and is a power series

$\vdots$

Examples: (1)

$$\exp(z) := \sum_{k=0}^{\infty} \frac{z^k}{k!} \qquad (\text{radius of convergence: } r = \infty)$$

$$\exp'(z) = \sum_{k=1}^{\infty} \frac{k \cdot z^{k-1}}{k!} = \sum_{k=1}^{\infty} \frac{z^{k-1}}{(k-1)!} = \sum_{m=0}^{\infty} \frac{z^m}{m!} = \exp(z)$$

(2)

$$\cos(z) := \sum_{k=0}^{\infty} (-1)^k \frac{z^{2\cdot k}}{(2\cdot k)!}$$

connection?

$$\exp(i \cdot z) = \sum_{k=0}^{\infty} \frac{\underbrace{(iz)^k}}{k!}, \qquad (iz)^k = \begin{cases} z^k, & k = 0, 4, 8, \ldots \\ i z^k, & k = 1, 5, 9, 13, \ldots \\ -z^k, & k = 2, 6, 10, \ldots \\ -i z^k, & k = 3, 7, 11, \ldots \end{cases}$$

$$\exp(-i \cdot z) = \sum_{k=0}^{\infty} \frac{(-iz)^k}{k!}, \qquad (-iz)^k = \begin{cases} z^k, & k = 0, 4, 8, \ldots \\ -i z^k, & k = 1, 5, 9, 13, \ldots \\ -z^k, & k = 2, 6, 10, \ldots \\ +i z^k, & k = 3, 7, 11, \ldots \end{cases}$$

$$\exp(i \cdot z) + \exp(-i \cdot z) = \sum_{m=0}^{\infty} (-1)^m \frac{z^{2m}}{(2m)!} \cdot 2 = 2 \cdot \cos(z)$$

$$\Longrightarrow \quad \cos(z) = \frac{1}{2}\Big(\exp(i \cdot z) + \exp(-i \cdot z)\Big)$$

$$\Longrightarrow \quad \cos'(z) = \frac{i}{2}\Big(\exp(i \cdot z) - \exp(-i \cdot z)\Big) = -\sin(z)$$