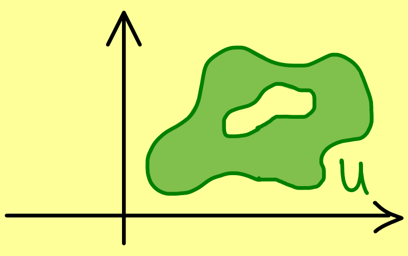
The Bright Side of Mathematics

Complex Analysis – Part 2

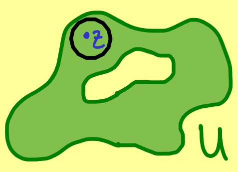
$f: \mathbb{C} \to \mathbb{C}$ differentiable at z_0?

domain can be any open set $U \subseteq \mathbb{C}$



Definition: $U \subseteq \mathbb{C}$ is called open if

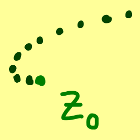
$$\forall z \in U \;\; \exists \varepsilon > 0 \;:\; B_\varepsilon(z) \subseteq U$$



Definition: $U \subseteq \mathbb{C}$ open, $z_0 \in U$. $f: U \to \mathbb{C}$ is called

(complex) differentiable at $z_0 \in U$ if

$$\lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0} \quad \text{exists.}$$



For all sequences $(z_n)_{n \in \mathbb{N}} \subseteq U \setminus \{z_0\}$ with $z_n \xrightarrow{n \to \infty} z_0$,

the sequence $\dfrac{f(z_n) - f(z_0)}{z_n - z_0}$ converges (to the same number).