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ON STEADY

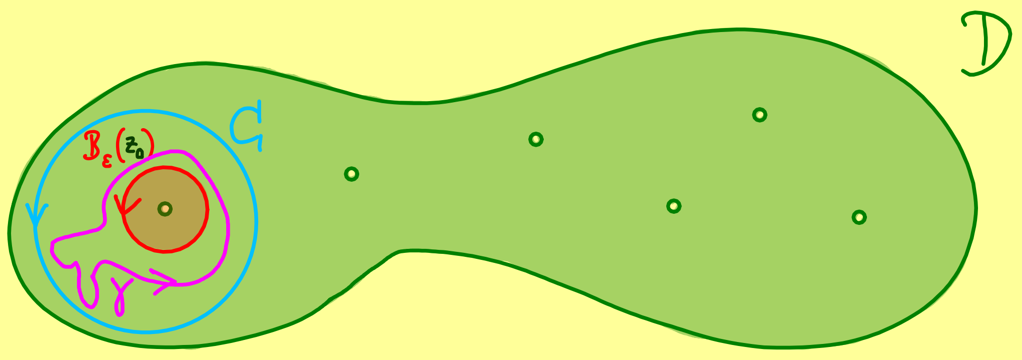
The Bright Side of Mathematics

Complex Analysis – Part 34

$f: D \to \mathbb{C}$

holomorphic

isolated singularity z_0

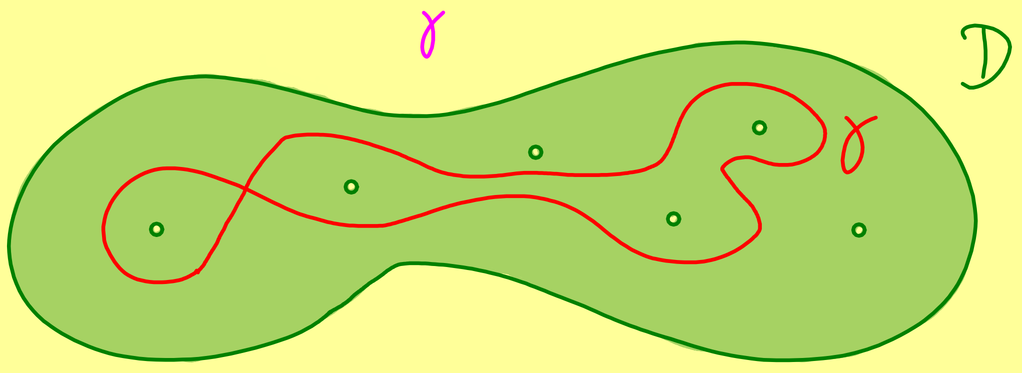


$$\mathrm{Res}(f, z_0) := \frac{1}{2\pi i} \oint_{\partial B_\varepsilon(z_0)} f(z)\, dz$$

$$\Rightarrow \quad 2\pi i \cdot \mathrm{Res}(f, z_0) = \oint_{\partial B_\varepsilon(z_0)} f(z)\, dz = \oint_{C} f(z)\, dz$$

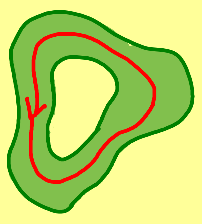
circle around z_0

$$\Rightarrow \quad \mathrm{wind}(\gamma, z_0) \cdot 2\pi i \cdot \mathrm{Res}(f, z_0) = \oint_{\gamma} f(z)\, dz$$



Residue theorem: $D \subseteq \mathbb{C}$ open domain, (connected) $f: D \to \mathbb{C}$ holomorphic, $z_1, z_2, \dots, z_n$ isolated singularities of f, $\gamma: [a,b] \to D$ closed curve with $\mathrm{Int}(\gamma) \subseteq D \cup \{z_1, z_2, \dots, z_n\}$.

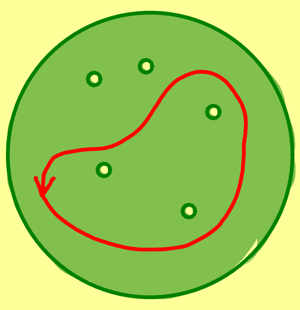
not allowed:

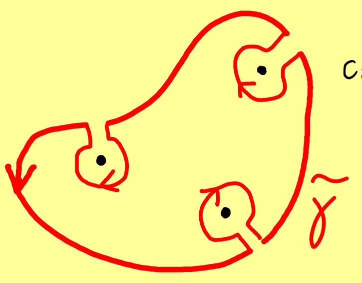


Then:

$$\oint_{\gamma} f(z)\, dz = \sum_{j=1}^{n} 2\pi i \cdot \mathrm{wind}(\gamma, z_j) \cdot \mathrm{Res}(f, z_j)$$

Proof: $\tilde{D} \subseteq \mathbb{C}$ open disc, $D = \tilde{D} \setminus \{z_1, z_2, \dots, z_n\}$.





Cauchy's theorem $\rightsquigarrow$ $\oint_{\tilde{\gamma}} f(z)\, dz = 0$