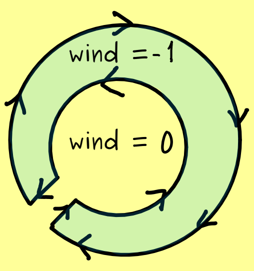
The Bright Side of Mathematics

Complex Analysis – Part 25

winding number: $\operatorname{wind}(\gamma, z_0) := \frac{1}{2\pi i} \oint_{\gamma} \frac{1}{z-z_0}\, dz$



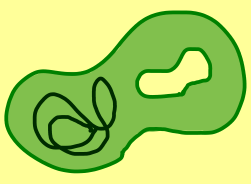
<u>Definition:</u> For $\gamma : [a,b] \longrightarrow \mathbb{C}$ closed :

$$\operatorname{Ext}(\gamma) := \left\{ z_0 \in \mathbb{C} \setminus \operatorname{Ran}(\gamma) \;\middle|\; \operatorname{wind}(\gamma, z_0) = 0 \right\}$$

$$\operatorname{Int}(\gamma) := \left\{ z_0 \in \mathbb{C} \setminus \operatorname{Ran}(\gamma) \;\middle|\; \operatorname{wind}(\gamma, z_0) \neq 0 \right\}$$

<u>Extending Cauchy's theorem:</u>

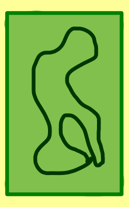
$f : D \longrightarrow \mathbb{C}$ holomorphic , γ closed, $\operatorname{Int}(\gamma) \cup \operatorname{Ran}(\gamma) \subseteq D$



$D =$ disc

$\overset{\text{part 23}}{\Longrightarrow} \oint_{\gamma} f(z)\, dz = 0$

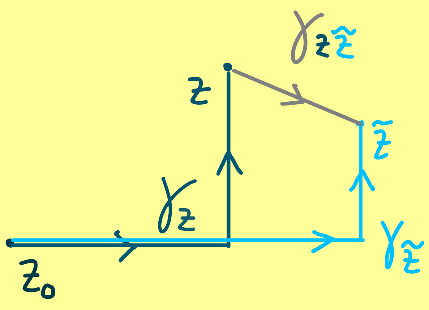
$D =$ rectangle



$\overset{\substack{\text{same proof}\\ \text{part 23}}}{\Longrightarrow} \oint_{\gamma} f(z)\, dz = 0$

proof needed:

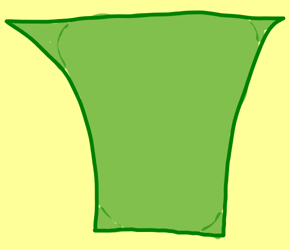
$F(z) := \int_{\gamma_z} f(\xi)\, d\xi$



$\oint_{\gamma_{\tilde{z}} + \gamma_z^- + \gamma_{z\tilde{z}}^-} f(z)\, dz \overset{\text{Goursat}}{=} 0$

works also: $D =$

, $D =$



<u>Cauchy's theorem (general version):</u>

$f : D \longrightarrow \mathbb{C}$ holomorphic , γ closed, $\operatorname{Int}(\gamma) \cup \operatorname{Ran}(\gamma) \subseteq D$

$\Longrightarrow \oint_{\gamma} f(z)\, dz = 0$

<u>Cauchy's theorem (for some domains):</u>

$f : D \longrightarrow \mathbb{C}$ holomorphic , $\gamma : [a,b] \longrightarrow D$ closed curve,

If $\left\{ \begin{array}{l} D \text{ convex} \quad \text{or} \\ D = \quad \text{or} \\ D \text{ star domains} \end{array} \right\}$ $\Longrightarrow \oint_{\gamma} f(z)\, dz = 0$