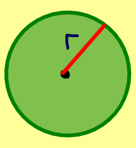
BECOME A MEMBER

ON STEADY

The Bright Side of Mathematics

Complex Analysis – Part 12

$f(z) = \sum_{k=0}^{\infty} a_k \cdot z^k$ → holomorphic on its open disc of convergence 

→ f' exists and is a power series

f'' exists and is a power series

$\vdots$

Examples: (1) $\exp(z) := \sum_{k=0}^{\infty} \frac{z^k}{k!}$ (radius of convergence: $r = \infty$)

$$\exp'(z) = \sum_{k=1}^{\infty} \frac{k \cdot z^{k-1}}{k!} = \sum_{k=1}^{\infty} \frac{z^{k-1}}{(k-1)!} = \sum_{m=0}^{\infty} \frac{z^m}{m!} = \exp(z)$$

(2) $\cos(z) := \sum_{k=0}^{\infty} (-1)^k \frac{z^{2\cdot k}}{(2\cdot k)!}$

connection?

$$\exp(i\cdot z) = \sum_{k=0}^{\infty} \frac{(iz)^k}{k!}, \quad (iz)^k = \begin{cases} z^k, & k = 0,4,8,\dots \\ i z^k, & k = 1,5,9,13,\dots \\ -z^k, & k = 2,6,10,\dots \\ -i z^k, & k = 3,7,11,\dots \end{cases}$$

$$\exp(-i\cdot z) = \sum_{k=0}^{\infty} \frac{(-iz)^k}{k!}, \quad (-iz)^k = \begin{cases} z^k, & k = 0,4,8,\dots \\ -i z^k, & k = 1,5,9,13,\dots \\ -z^k, & k = 2,6,10,\dots \\ +i z^k, & k = 3,7,11,\dots \end{cases}$$

$$\exp(i\cdot z) + \exp(-i\cdot z) = \sum_{m=0}^{\infty} (-1)^m \frac{z^{2m}}{(2m)!} \cdot 2 = 2 \cdot \cos(z)$$

$$\Longrightarrow \quad \cos(z) = \frac{1}{2}\Big(\exp(i\cdot z) + \exp(-i\cdot z)\Big)$$

$$\Longrightarrow \quad \cos'(z) = \frac{i}{2}\Big(\exp(i\cdot z) - \exp(-i\cdot z)\Big) = -\sin(z)$$