The Bright Side of Mathematics

Complex Analysis – Part 10

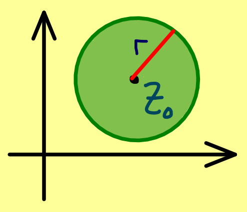
Definition: A sequence of functions $f_n : U \to \mathbb{C}$ $\quad (n \in \mathbb{N})$

is uniformly convergent to $f : U \to \mathbb{C}$

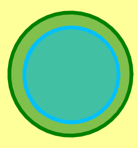
if $\underbrace{\| f_n - f \|_\infty}_{:= \sup_{z \in U} | f_n(z) - f(z) |} \xrightarrow{n \to \infty} 0$.

Result for power series: Let $f : B_r(z_0) \to \mathbb{C}$, $f(z) = \sum_{k=0}^{\infty} a_k \cdot (z - z_0)^k$

be a power series with radius of convergence $r > 0$.



Then: (1) $\sum_{k=0}^{\infty} a_k \cdot (z - z_0)^k$ is uniformly convergent on $\overline{B_c(z_0)}$ with $c < r$



(sequence of functions $f_n : \overline{B_c(z_0)} \to \mathbb{C}$, $f_n(z) = \sum_{k=0}^{n} a_k \cdot (z - z_0)^k$ is uniformly convergent)

(2) $\sum_{k=1}^{\infty} a_k \cdot k (z - z_0)^{k-1}$ is uniformly convergent on $\overline{B_c(z_0)}$ with $c < r$

(sequence of functions $f_n' : \overline{B_c(z_0)} \to \mathbb{C}$, $f_n'(z) = \sum_{k=0}^{n} a_k \cdot k (z - z_0)^{k-1}$ is uniformly convergent)

(3) f is complex differentiable with $f'(z) = \sum_{k=1}^{\infty} a_k \cdot k (z - z_0)^{k-1}$