

Calculating the kernel of matrix

(an example)

$$A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 3 & 1 & 4 & 4 \\ 4 & -4 & 0 & 8 \end{pmatrix} \in \mathbb{R}^{3 \times 4}$$

$$\ker(A) := \{x \in \mathbb{R}^4 \mid Ax = 0\}$$

$$\ker(A) = \ker \begin{pmatrix} 1 & 1 & 2 & 1 \\ 3 & 1 & 4 & 4 \\ 4 & -4 & 0 & 8 \end{pmatrix} \begin{array}{l} \leftarrow \text{II} - 3\text{I} \\ \leftarrow \text{III} - 4\text{I} \end{array}$$

$$= \ker \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & -2 & -2 & 1 \\ 0 & -8 & -8 & 4 \end{pmatrix} \leftarrow \text{III} - 4\text{II}$$

$$= \ker \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & -2 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$-2x_2 - 2x_3 + x_4 = 0$
 $\Rightarrow x_2 = -x_3 + \frac{1}{2}x_4$

$x_1 + x_2 + 2x_3 + x_4 = 0$
 $= -x_3 + \frac{1}{2}x_4 + 2x_3 + x_4 = 0$
 $x_1 = -x_3 - \frac{3}{2}x_4$

$$= \left\{ \begin{pmatrix} -\alpha - \frac{3}{2}\beta \\ -\alpha + \frac{1}{2}\beta \\ \alpha \\ \beta \end{pmatrix} \mid \alpha, \beta \in \mathbb{R} \right\}$$

$$= \left\{ \alpha \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + \beta \cdot \begin{pmatrix} -\frac{3}{2} \\ \frac{1}{2} \\ 0 \\ 1 \end{pmatrix} \mid \alpha, \beta \in \mathbb{R} \right\}$$

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basis of this $\ker(A)$