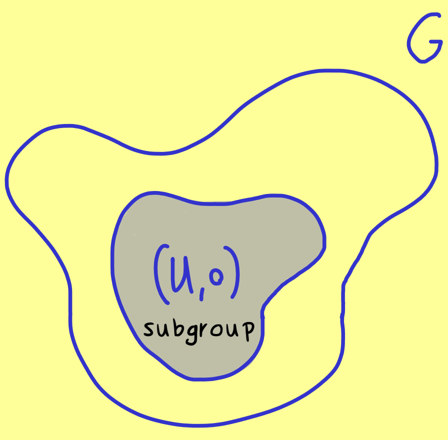
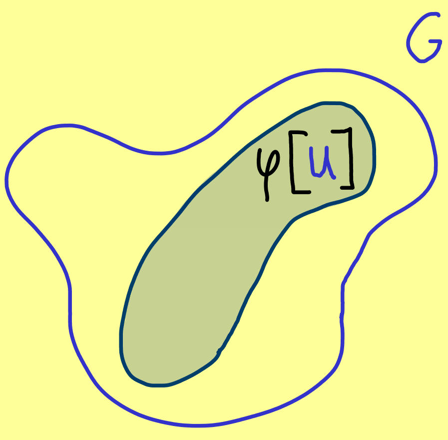
Algebra - Part 13



$\varphi: G \to G$

group homomorphism
=
group endomorphism



Important case: inner automorphisms: $\varphi: G \to G$ group homomorphism that can be written as $\varphi(x) = g\,x\,g^{-1}$

inner: φ is represented by an inner element

automorphisms: endomorphism + isomorphism

isomorphism = (bijective and homomorphism in both directions)

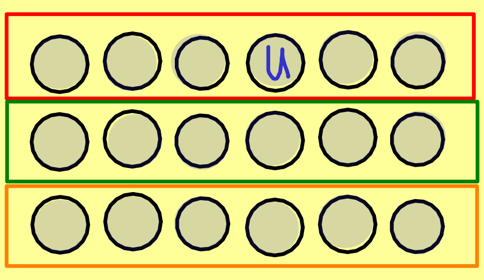
We already know: $U \subseteq G$ subgroup $\Rightarrow$ $\varphi[U]$, $\varphi^{-1}[U]$ subgroups

Definition: Two subgroups $U, V \subseteq G$ are called conjugate subgroups if there is an element $g \in G$: $V = gUg^{-1} := \{ g\,u\,g^{-1} \mid u \in U \}$

$= \varphi[U]$ for $\varphi: G \to G$, $\varphi(x) = g\,x\,g^{-1}$

Remember: This defines an equivalence relation on the set of subgroups of G.

Hence:



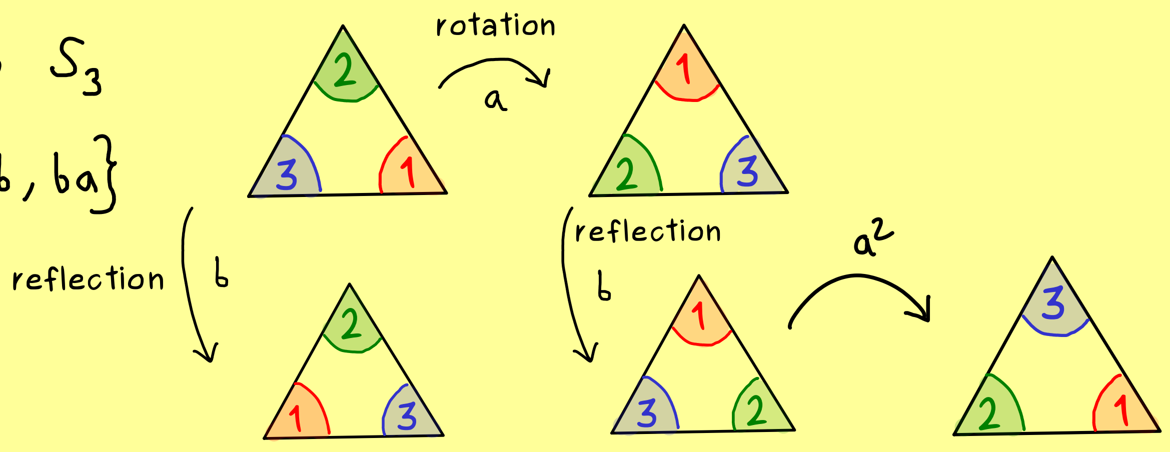
$[U] := \{ gUg^{-1} \mid g \in G \}$

equivalence class

Trivial for abelian groups: $gUg^{-1} = \{ u\,g\,g^{-1} \mid u \in U \} = U$

Example: Symmetric group S_3

$S_3 = \{ e, a, b, a^2, ab, ba \}$



$U = \{e, b\}$ conjugate subgroups

$a\,U\,a^{-1} = \{ e, \underbrace{aba^2}_{ba} \} = \{ e, ba \}$

$a^2\,U\,(a^2)^{-1} = \{ e, \underbrace{a^2 b a}_{ab} \} = \{ e, ab \}$

$ab\,U\,(ab)^{-1} = \{ e, a\underbrace{bb}_{=e}(ab) \} = \{ e, ba \}$

$ba\,U\,(ba)^{-1} = \{ e, ba\underbrace{b(ba)}_{e} \} = \{ e, ab \}$

$b\,U\,b^{-1} = e\,U\,e^{-1} = U$