

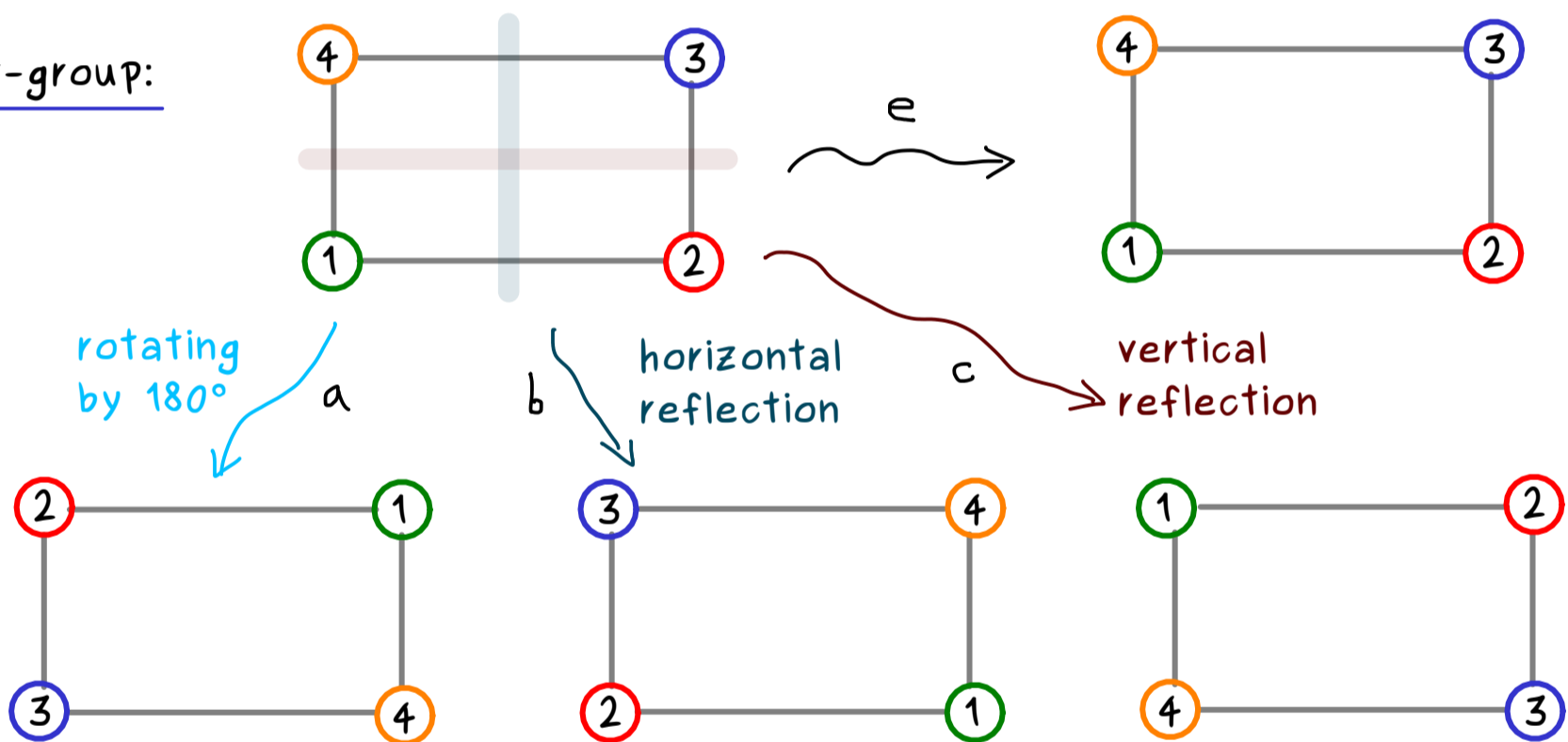
Algebra - Part 11

Recall subgroups: $(G, \circ) \rightsquigarrow H \subseteq G, (H, \circ) \text{ group} \rightsquigarrow H \text{ subgroup of } G$
 $\rightsquigarrow H \leq G$

Proposition: (G, \circ) group, $H \subseteq G$ non-empty subset.

$$H \leq G \iff \begin{cases} a \circ b \in H & \text{for all } a, b \in H \\ a^{-1} \in H & \text{for all } a \in H \end{cases}$$

Klein four-group:



o	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

\rightsquigarrow associativity ✓

(G, \circ) with $G = \{e, a, b, c\}$ and \circ satisfying the table above defines the so-called Klein four group, called K_4 .

Proposition: Let (G, \circ) be a group with $\text{ord}(G) < \infty$, $H \subseteq G$ be a non-empty subset.

Then: $H \leq G \iff a \circ b \in H$ for all $a, b \in H$

Proof: (\implies) ✓ (\impliedby) (H, \circ) semigroup of finite order and both cancellation properties hold

$$\left(\begin{array}{l} a \circ x = a \circ y \implies x = y \\ x \circ b = y \circ b \implies x = y \end{array} \right)$$

part 6

$\implies (H, \circ)$ is a group

□

Example: $G = \{e, a, b, c\}$ Klein four-group.

subgroups: $H_1 = \{e\}$, $H_2 = \{e, a\}$, $H_3 = \{e, b\}$, $H_4 = \{e, c\}$, $H_5 = G$

\rightsquigarrow we have 5 subgroups