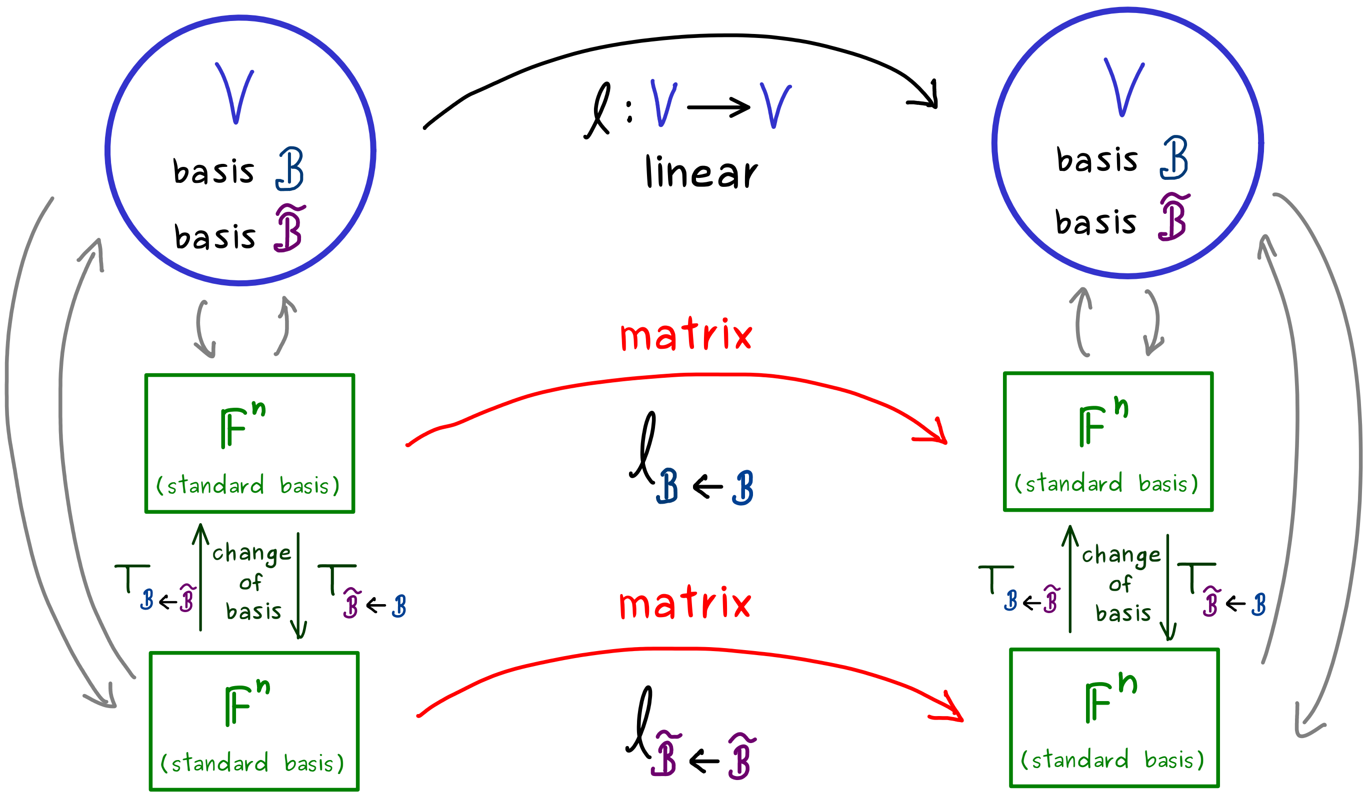
Abstract Linear Algebra - Part 30



We have:

$$\ell_{\widetilde{\mathcal{B}} \leftarrow \widetilde{\mathcal{B}}} = T_{\widetilde{\mathcal{B}} \leftarrow \mathcal{B}} \; \ell_{\mathcal{B} \leftarrow \mathcal{B}} \; T_{\mathcal{B} \leftarrow \widetilde{\mathcal{B}}}$$

$$\tilde{A} = T^{-1} \, A \, T$$

Definition: A matrix $\tilde{A} \in \mathbb{F}^{n \times n}$ is called similar to a matrix $A \in \mathbb{F}^{n \times n}$ if there is an invertible $T \in \mathbb{F}^{n \times n}$ such that:

$$\tilde{A} = T^{-1} A T .$$

We write: $\tilde{A} \approx A$.

Remark: $\approx$ defines an equivalence relation on $\mathbb{F}^{n \times n}$:

(1) reflexive: $A \approx A$ for all $A \in \mathbb{F}^{n \times n}$

(2) symmetric: $A \approx B \implies B \approx A$ for all $A, B \in \mathbb{F}^{n \times n}$

(3) transitive: $A \approx B \wedge B \approx C \implies A \approx C$ for all $A, B, C \in \mathbb{F}^{n \times n}$

Easy to see: $A \approx B \implies A \sim B$

Example: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ but $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \not\approx \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$T^{-1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\approx$ is characterized by the so-called Jordan normal form