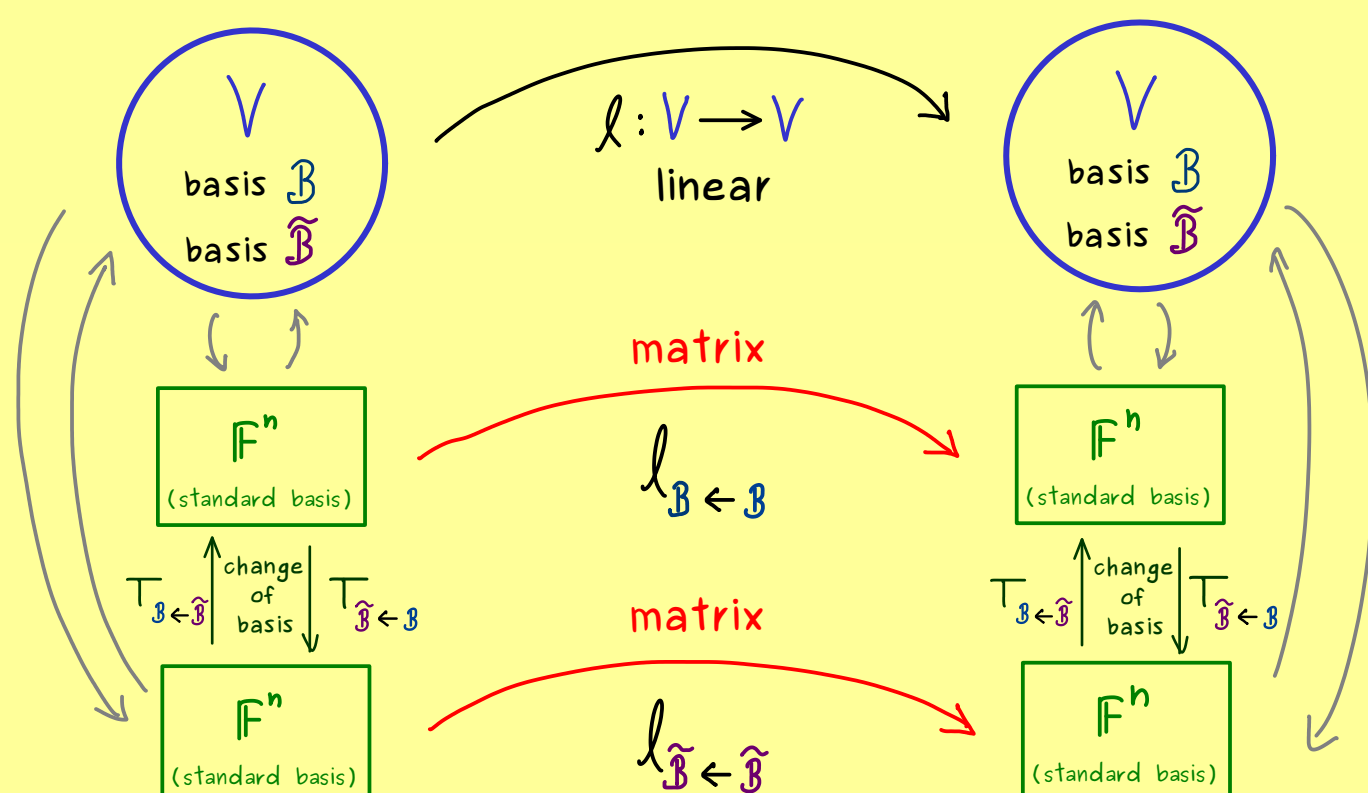




Abstract Linear Algebra - Part 30



We have:

$$\begin{aligned} l_{\tilde{\mathcal{B}} \leftarrow \tilde{\mathcal{B}}} &= T_{\tilde{\mathcal{B}} \leftarrow \mathcal{B}} l_{\mathcal{B} \leftarrow \mathcal{B}} T_{\mathcal{B} \leftarrow \tilde{\mathcal{B}}} \\ \parallel & \quad \parallel \quad \parallel \quad \parallel \\ \tilde{A} &= T^{-1} A T \end{aligned}$$

Definition: A matrix $\tilde{A} \in \mathbb{F}^{n \times n}$ is called similar to a matrix $A \in \mathbb{F}^{n \times n}$ if there is an invertible $T \in \mathbb{F}^{n \times n}$ such that:

$$\tilde{A} = T^{-1} A T.$$

We write: $\tilde{A} \approx A$.

Remark: \approx defines an equivalence relation on $\mathbb{F}^{n \times n}$:

(1) reflexive: $A \approx A$ for all $A \in \mathbb{F}^{n \times n}$

(2) symmetric: $A \approx B \Rightarrow B \approx A$ for all $A, B \in \mathbb{F}^{n \times n}$

(3) transitive: $A \approx B \wedge B \approx C \Rightarrow A \approx C$ for all $A, B, C \in \mathbb{F}^{n \times n}$

Easy to see: $A \approx B \Rightarrow A \sim B$

Example: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ but $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \not\approx \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$
 $\hookrightarrow T^{-1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

\approx is characterized by the so-called Jordan normal form