Abstract Linear Algebra - Part 28

Fact: $\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 3 & -2 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ -1 & 0 & 1 & 1 \end{pmatrix}$ are <u>different</u> but

they describe the <u>same</u> linear map $\ell : \mathcal{P}_3(\mathbb{R}) \longrightarrow \mathcal{P}_2(\mathbb{R})$, $\ell(p) = p'$

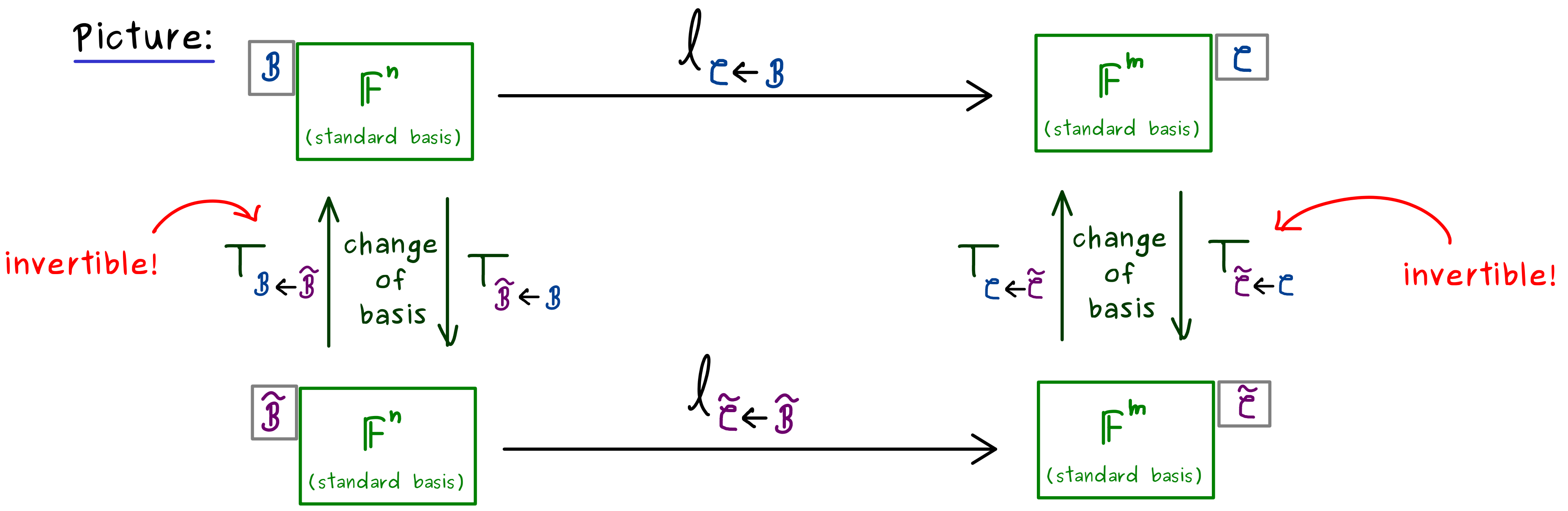
with respect to different bases.

Question: $\ell : V \longrightarrow W$ linear, $A = \ell_{\mathcal{C} \leftarrow \mathcal{B}} \in \mathbb{F}^{m \times n}$.

For another $\tilde{A} \in \mathbb{F}^{m \times n}$, can we find bases such that $\tilde{A} = \ell_{\tilde{\mathcal{C}} \leftarrow \hat{\mathcal{B}}}$?

If YES!, then we say A and $\tilde{A}$ are <u>equivalent</u>.

Picture:



Definition: A matrix $\tilde{A} \in \mathbb{F}^{m \times n}$ is called <u>equivalent to a matrix</u> $A \in \mathbb{F}^{m \times n}$

if there are invertible matrices $S \in \mathbb{F}^{m \times m}$, $T \in \mathbb{F}^{n \times n}$, such that:

$$\tilde{A} = S\,A\,T.$$

We write: $\tilde{A} \sim A$

Remark: $\sim$ defines an equivalence relation on $\mathbb{F}^{m \times n}$:

(1) reflexive: $A \sim A$ for all $A \in \mathbb{F}^{m \times n}$

(2) symmetric: $A \sim B \Rightarrow B \sim A$ for all $A, B \in \mathbb{F}^{m \times n}$

(3) transitive: $A \sim B \wedge B \sim C \Rightarrow A \sim C$ for all $A, B, C \in \mathbb{F}^{m \times n}$