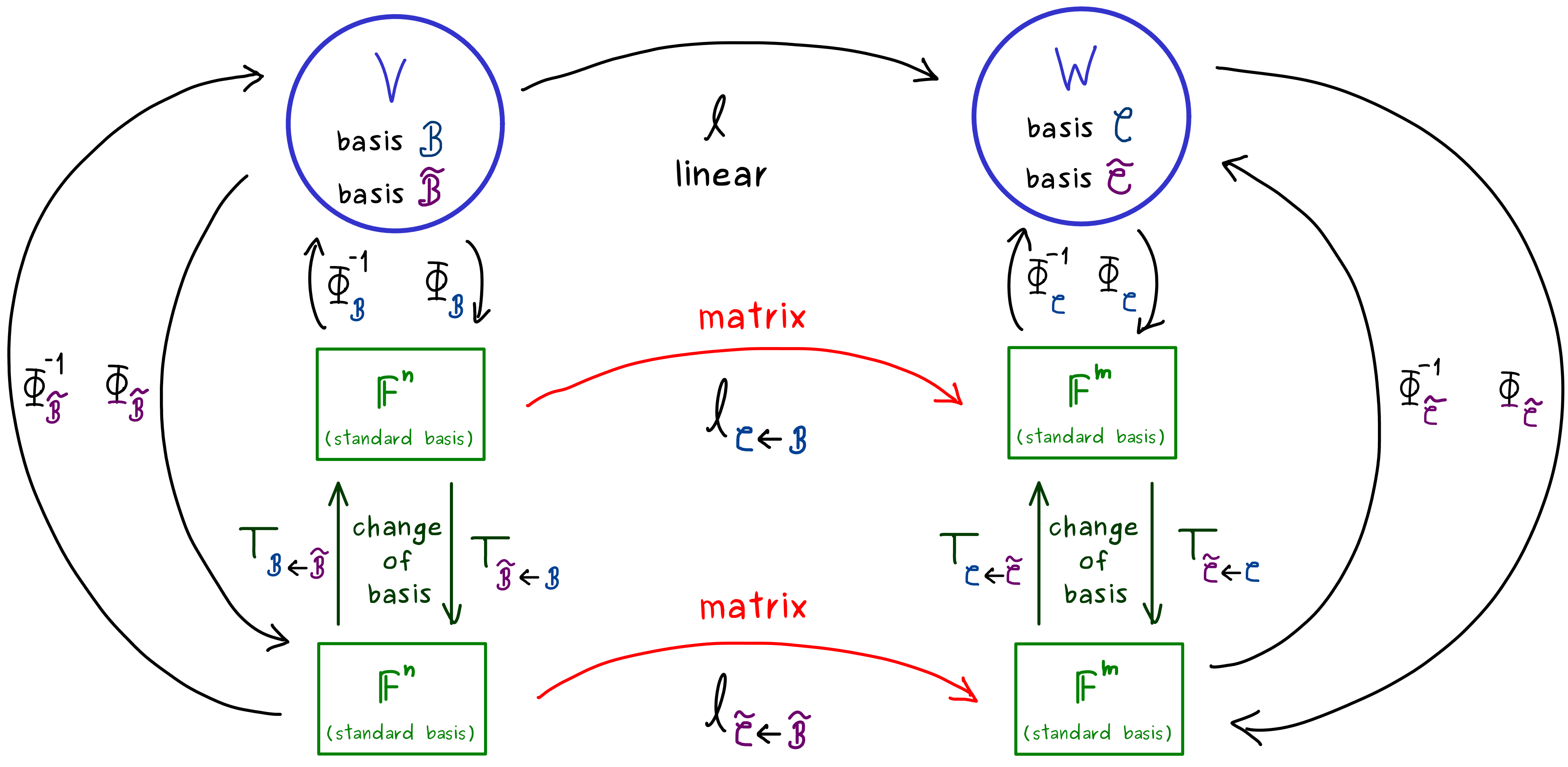
Abstract Linear Algebra – Part 27



Result:

$$\boxed{\ell_{\tilde{\mathcal{C}}\leftarrow\tilde{\mathcal{B}}} = T_{\tilde{\mathcal{C}}\leftarrow\mathcal{C}}\,\ell_{\mathcal{C}\leftarrow\mathcal{B}}\,T_{\mathcal{B}\leftarrow\tilde{\mathcal{B}}}}$$

Example: $\ell : \mathcal{P}_3(\mathbb{R}) \longrightarrow \mathcal{P}_2(\mathbb{R})$, $\ell(p) = p'$ linear map!

$\mathcal{B} = (m_3, m_2, m_1, m_0)$ $\mathcal{C} = (m_2, m_1, m_0)$

$\tilde{\mathcal{B}} = (2m_3 - m_1,\ m_2 + m_0,\ m_1 + m_0,\ m_1 - m_0)$, $\tilde{\mathcal{C}} = (m_2 - \frac{1}{2}m_1,\ m_2 + \frac{1}{2}m_1,\ m_0)$

matrix representation: $\ell_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

change-of-basis matrices: $T_{\mathcal{B}\leftarrow\tilde{\mathcal{B}}} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$

$$T_{\mathcal{C}\leftarrow\tilde{\mathcal{C}}} = \begin{pmatrix} 1 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{inverse}} T_{\tilde{\mathcal{C}}\leftarrow\mathcal{C}} = \begin{pmatrix} \frac{1}{2} & -1 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\ell_{\tilde{\mathcal{C}}\leftarrow\hat{\mathcal{B}}} = T_{\tilde{\mathcal{C}}\leftarrow\mathcal{C}}\, \ell_{\mathcal{C}\leftarrow\mathcal{B}}\, T_{\mathcal{B}\leftarrow\hat{\mathcal{B}}} = \begin{pmatrix} \frac{1}{2} & -1 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & -1 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ -1 & 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -2 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ -1 & 0 & 1 & 1 \end{pmatrix}$$